Advances in Statistical Compact Modeling

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Overview

- Statistical Compact Modeling and use model
- Method is available as GUI based MunEDA WiCkeD App “Statistical Fit”
- Data normalization for fitting
- Excluding mismatch effects by calculation from PCM measurements
- Helpful input data consistency checks
- Validation of results

Task and implementation idea

Target User Group

- CAD Modeling & Characterization engineers

Task

- Improve consistency of statistical spice models according to variations and correlations in order to …
- Improve the variation of analog device behavior in statistical (Monte Carlo) simulations

Implementation idea

- Introducing correlations with statistical spice model card parameters
- The standard deviation is taken from PCM specification
- The correlations between PCMs can be extracted from PCM measurements

General use model

PCM specification
source of the PCM standard deviation

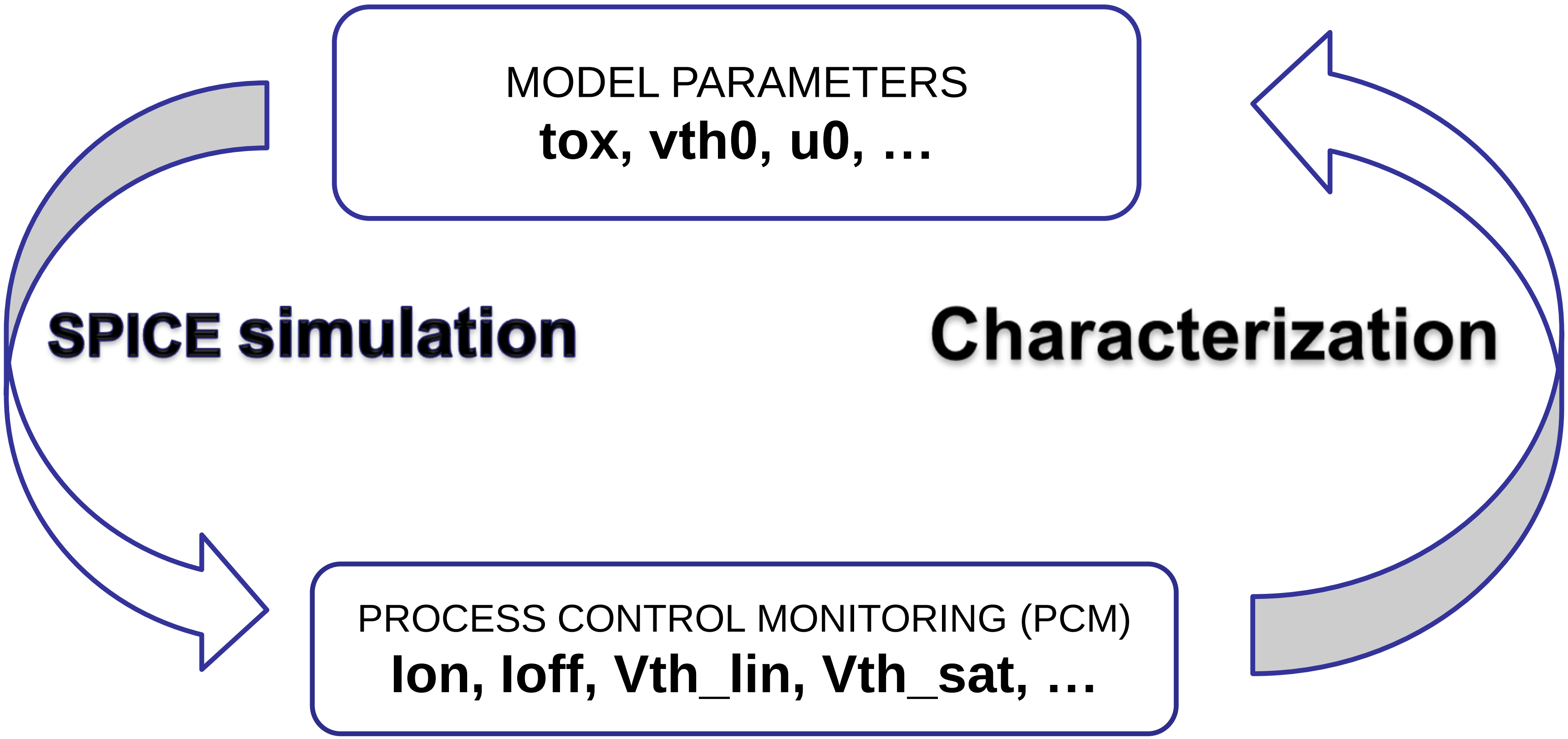
PCM spice simulation
PCM testbench with initial statistical models

PCM data sets (measurements)
source of the PCM correlation

Elaborating
WiCkeD "Statistical Fit"

Improved device statistics models

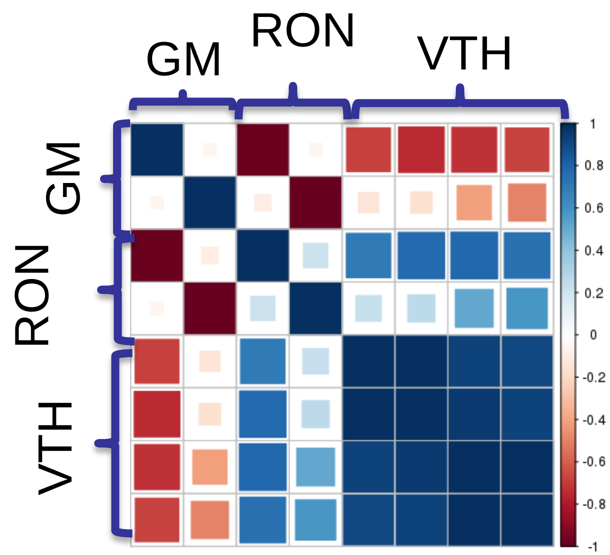
Relation between model parameters and PCM re-simulation

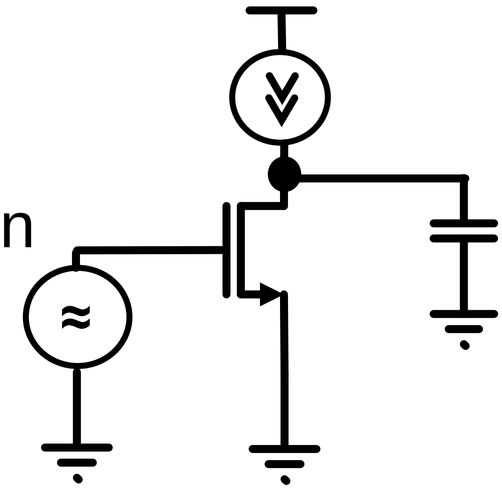


Correlations between model parameters (vth0, u0, tox, …) are necessary to reproduce correct variance and correlation in process MC for analog circuits

Why is the correlation between PCMs so important for analog design?

- Common source single stage amplifier
- Low frequency gain: $A_v = G_m \cdot R_{on}$
- There is strong negative correlation between G_m and R_{on}:





- The MC model must reproduce this correlation in order to correctly simulate the variance of the gain

Typical problem with modeling of the correlation of PCMs

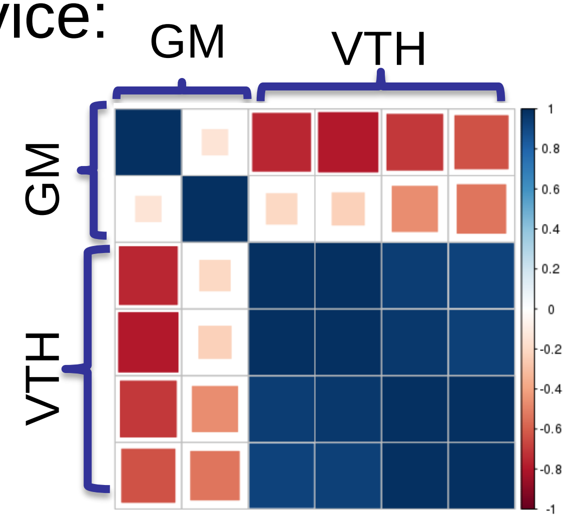
- Three typical model parameters for process MC:
  - tox – oxide thickness
  - u0 – mobility
  - vth0 – threshold voltage

$I_{ds} \approx u0/tox \cdot (V_{gs} - vth0 - V_{ds}/2)$

$GM \approx u0/tox$

$VTH\_LIN \approx vth0$

- Consider two PCMs from the same device:
  - GM – maximum gm
  - VTH_LIN – extrapolated Vth
- The variance of Gm:
  - ≈ 60% dependence on u0
  - ≈ 40% dependence on tox
- The variance of VTH_LIN:
  - ≈ 100% dependence on vth0



tox, **u0** and **vth0** must be correlated to see a correlation of **GM** and **VTH_LIN** in simulation

Linear dependence of PCM variation on Model parameter variations

- Assumption of a linear dependency between PCM and model parameters

$$\mathbf{\Delta P = S \cdot \Delta M}$$

- ΔP – variation of PCM
- S – sensitivity of PCMs w.r.t. model parameters M (by simulation)
- ΔM – variation of model parameters

$$\mathbf{cov(P) = S \cdot cov(M) \cdot S^T}$$

- Least-Squares fit:

$$\min_X \|\underbrace{cov(P)}_{\text{measured}} - \underbrace{S}_{\text{simulated}} \cdot X \cdot S^T\|^2$$

Covariance matrix of PCM

- The variances of target PCMs are defined by specification:
  - **σ(P) are defined from technology , e.g. (USL-LSL)/9**
- The correlation of PCMs are measured by the FAB:
  - **corr(P) are calculated from wafer**
- Covariance matrix of PCM are constructed from variances and correlations of PCMs:

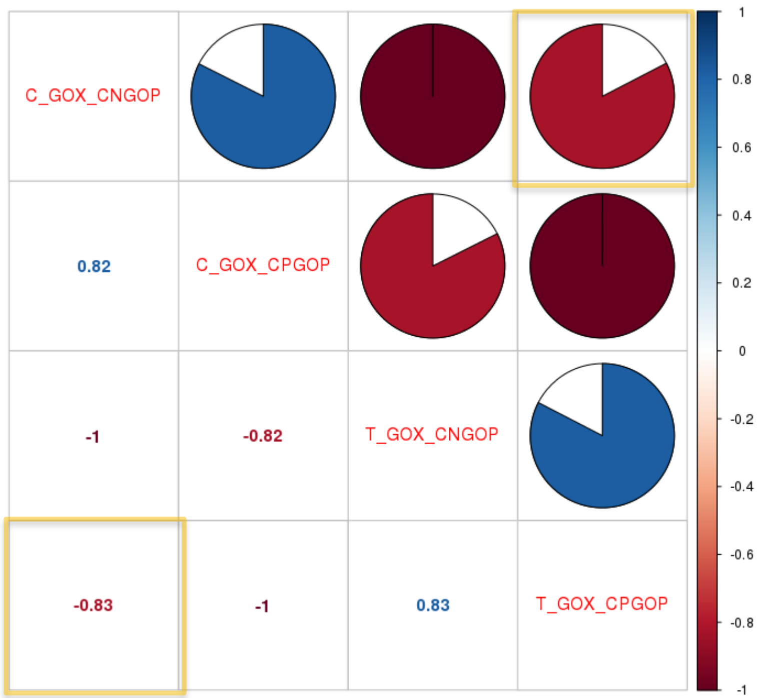
$$\mathrm{cov}(P) = \sigma(P) \cdot \mathrm{corr}(P) \cdot \sigma(P)$$

Capacitor example

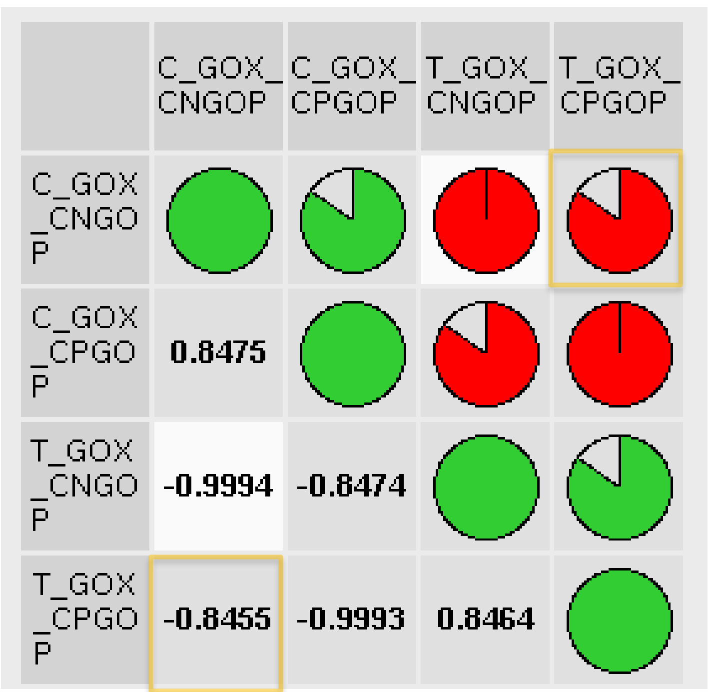
- E.g. two different types of capacitors:
  - Modelled with two parameters and one correlation factor
    - toxn
    - toxp

    } cc_toxn_toxp

Correlation matrix of PCM measurements



Simulation by Monte-Carlo



cc(T_GOX_CPGOP,C_GOX_CNGOP)=-0.83

cc(T_GOX_CPGOP,C_GOX_CNGOP)=-0.85

MunEDA WiCkeD App "Statistical Fit"

- GUI tool to create Statistical Models considering PCM correlations for analog transistor behavior that reproduce long-term process variation
- MunEDA WiCkeD is applied to the PCM netlists
  - Simulation setup with initial variation model of spice models
  - Selection of statistical model card parameters to consider
- Additional inputs of "Statistical Fit"
  - PCM measurements (data sets) for correlations target
  - PCM specification to determine the standard deviation of the statistical model card parameters as well
- Results of "Statistical Fit"
  - Set of model parameter sigmas (global MC parameters) and
  - correlations between them
- Benefit: More realistic sigmas / correlations of MC parameters
  - Safer verification
  - Less pessimism in design → better utilization of process technology

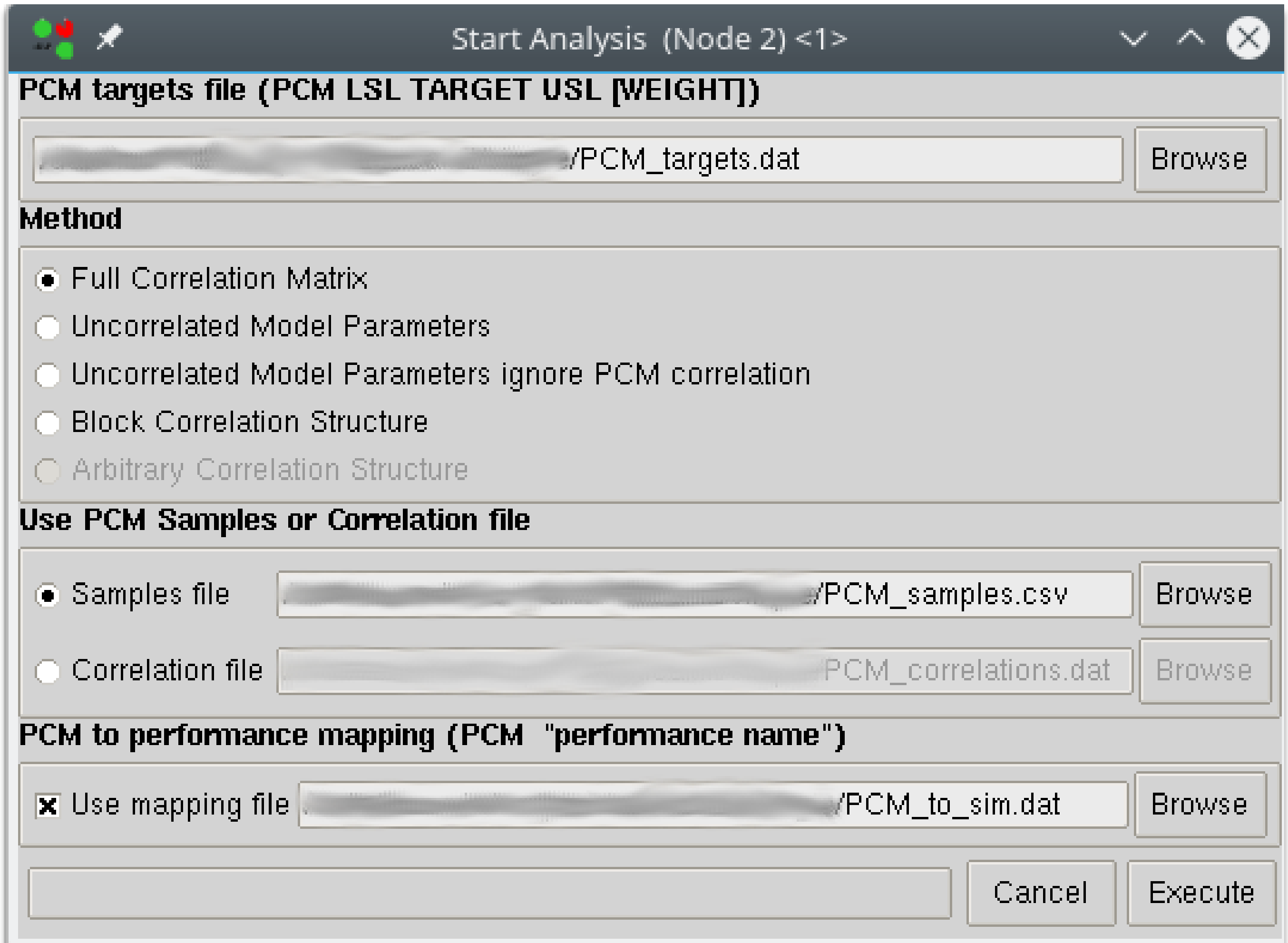
Screenshot of the WiCkeD App “Statistical Fit”

Selection of file with target sigmas

Selection of strategy

Selection of PCM data file or Correlation file

Mapping PCM name to measurement



Nonlinear PCMs with respect to the statistical model parameters

- Assumption of a linear dependency between PCM and model parameters

$$\Delta P = S \cdot \Delta M$$

ΔP variation of PCM

S sensitivity of PCMs w.r.t. model parameters M (by simulation)

ΔM variation of model parameters

- Sample PCMs are not linear; transform them to alternative:
  - COX – measurements of capacitance $\Rightarrow$ TOX – oxide thickness
  - RON – measurements of on-resistance $\Rightarrow$ GON – on-conductance
  - ILEAK – leakage current $\Rightarrow$ log(ILEAK) – logarithm of ILEAK

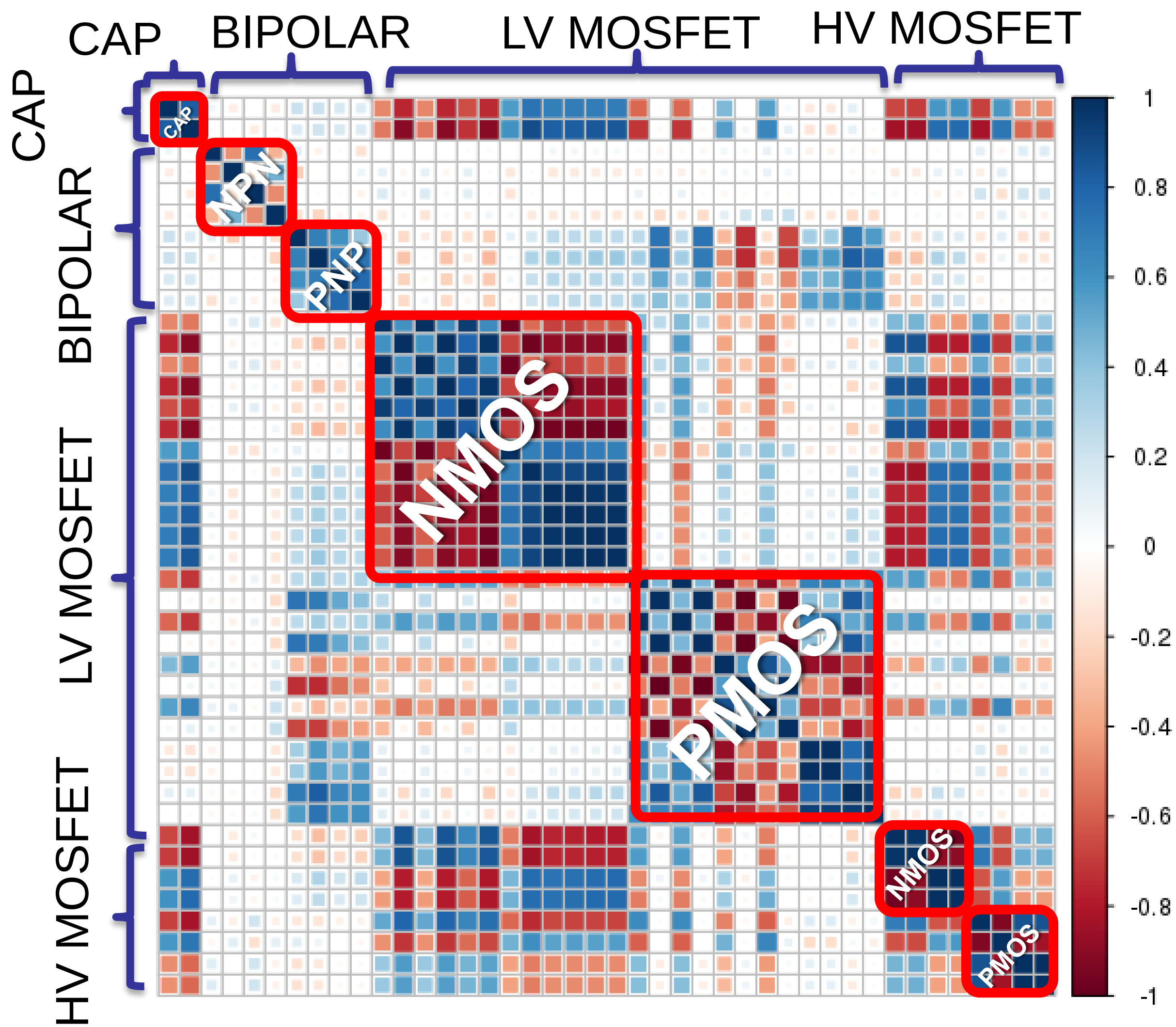
- In order to include second order nonlinearities into the dependence of PCMs on parameter variation, consider, “quadratic BPV” *

* I. Stevanovic, C.C.McAndrew, „Quadratic Backward Propagation of Variance for Nonlinear Statistical Circuit Modeling“, IEEE on CAD, V.28,No.9, 2009

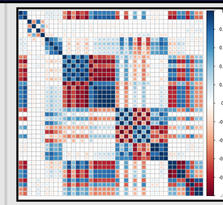
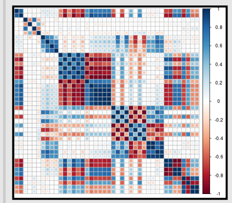
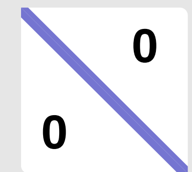
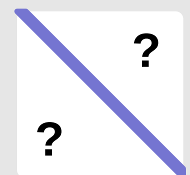
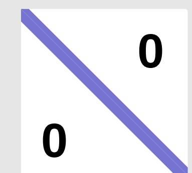
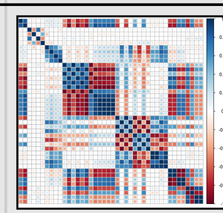
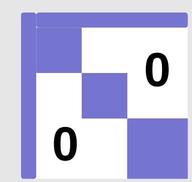
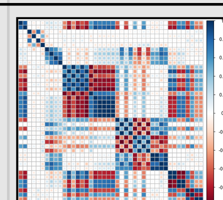
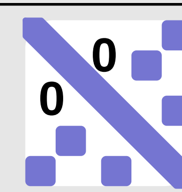
Sample structure of correlation matrix of PCMs (Measurement)

Typical correlation matrix of PCMs (Long-term data – 3 years of meas)

- DEVICES
- CAPACITOR
  - Different types
- BIPOLAR
  - NPN
  - PNP
- Low Voltage MOSFET
  - NMOS (diff geom)
  - PMOS (diff geom)
- High Voltage MOSFET
  - NMOS (diff geom)
  - PMOS (diff geom)



Strategies for Statistical Compact Modeling

Method	Purpose	Matrix of PCMs		Matrix of Model Parameters	
Full Correlation Matrix	Mathematical approach without physical distinguishes of model parameters	Full Covariance		Full Covariance	
Uncorrelated Model Parameters	Pay attention on correlation of PCMs, but produce uncorrelated model parameters matrix	Full Covariance		Only diagonal elements	
Uncorrelated Model Parameters ignore PCM correlation	Unknown correlation between PCMs, preliminary estimation of model parameters' sigmas	Only diagonal elements		Only diagonal elements	
Block Correlation Structure	Known correlation structure of model parameters, take into account correlation between PCMs	Full Covariance		Block-wise model parameters	
Arbitrary Correlation Structure	Know correlation structure of model parameters, but not block wise	Full Covariance		Arbitrary structure	

Normalization for least square fit

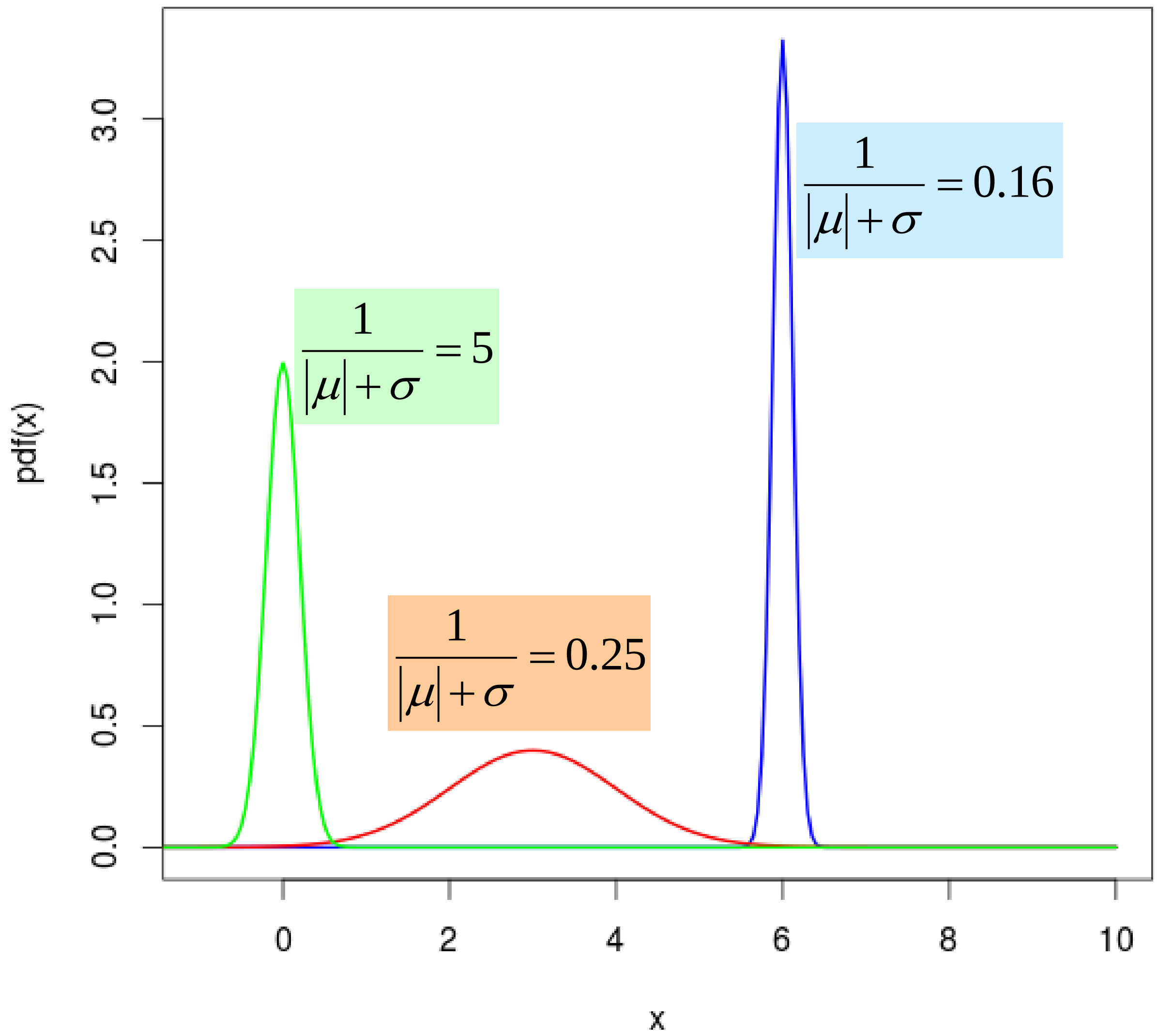
- PCMs have different orders of magnitude (kΩ, μA, …).
  Normalization needed to avoid large values dominating.
- PCMs have different relative variation σ/μ.
  Normalization needed to reduce weights of PCMs with $\sigma/|\mu| \ll 1$

Normalization factor for every PCM: $$\frac{1}{\sigma}\cdot\frac{\sigma/|\mu|}{1+\sigma/|\mu|}=\frac{1}{|\mu|+\sigma}$$

– i.e. instead of minimizing the difference in σ^2,
  we minimize the difference in $\sigma^2/(|\mu|+\sigma)^2$.

$\min_X \| N \cdot (\mathrm{cov}(P) - S \cdot X \cdot S^T) \cdot N \|^2$ with $N = \mathrm{diag}(1/(|\mu|+\sigma))$

Weight comparison of three example PCM



- PCM with smaller σ/μ are weighted less
- Works for μ=0
- Works for σ=0

Removing of mismatch effects from the target PCM covariance matrix

- The PCM measurements contain both process and mismatch effects. But, the correlation shall be just generated for the process variation
- Mismatch can be removed via the covariance matrix

$$\text{cov(P, process)} = \text{cov(P, measured)} - \text{cov(P, mismatch)}$$

- The mismatch covariance matrix is determined assuming a linear dependency between PCM and mismatch parameters

$$\mathbf{\Delta P = S_{mm} \cdot \Delta MM}$$

ΔP	variation of PCMs
S_{mm}	sensitivity of PCMs w.r.t. mismatch parameters (simulation)
ΔMM	variation of mismatch spice model parameters

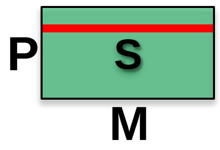
using

$$\text{cov(P,mismatch)} = S_{mm} \cdot \text{cov(mm)} \cdot S_{mm}^T$$

- cov(mm) is the covariance matrix of the mismatch model card parameters that are mm ~ $N(0,1^2)$ distributed. So, cov(mm) is typically an identity matrix.

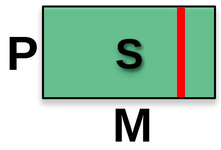
Input data consistency checks and improved data cleaning

1. Detection of PCMs with a low/no sensitivity towards the process parameters
   – There is no process parameter that can be used to model the variation of that PCMs
   – Case can be detected in the sensitivity matrix as (close to) zero row(s)



   ➔ User to add the missing process parameters
   ➔ To continue, those PCMs are excluded from further calculations

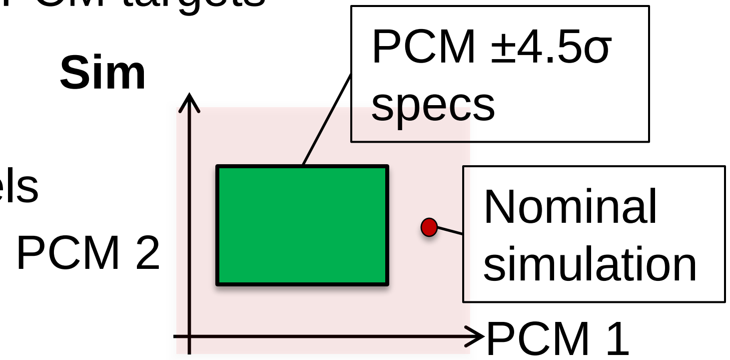
2. Detection of process parameters with a low/no sensitivity towards the PCMs
   – Without a sensitivity to PCMs, the statistics of those process parameters cannot be determined
   – Case can be detected in the sensitivity matrix as (close to) zero column(s)



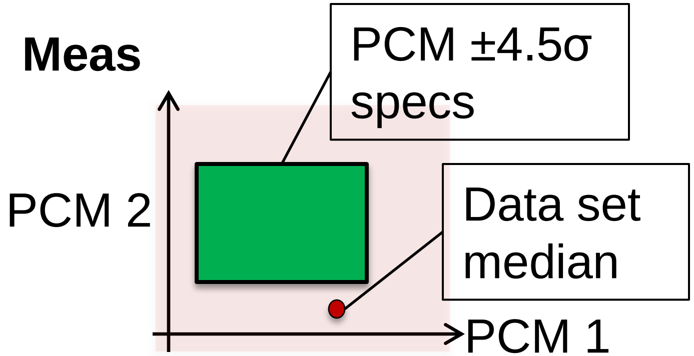
   ➔ User to pick the correct process parameter set
   ➔ To continue, such process parameters are excluded from further calculations

Input data consistency checks and improved data cleaning

3. Detection of PCMs nominal simulation outside of the PCM specification
   - The nominal PCM simulation does not fit to the PCM targets
   - Can be detected comparing the PCM nominal simulation with the PCM targets
   - ➔ Adjust the nominal simulation in the spice models
   - ➔ To continue, those PCMs are excluded from the calculations



4. Detection of PCM measurement median outside of the PCM specification
   - The PCM median of the measurements does not fit to the PCM specification
   - Can be detected comparing the median with the PCM targets
   - ➔ Verify that the preprocessing steps of the PCM measurements are correct
   - ➔ Correct the PCM targets and specifications



Input data consistency checks and improved data cleaning

5. Removing outliers from PCM measurements
   - Values outside of median ±5.5σ are considered as outliers
   - PCM data sets have many missing values
     - Do not remove complete data sets ➔ would reduce the number of data sets significantly
     - Instead mark outliers as missing value

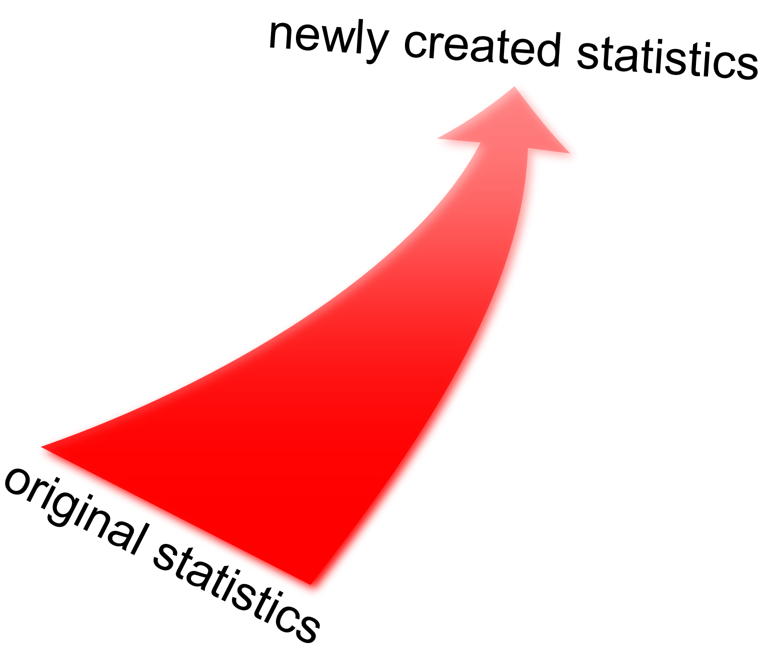
   ➔ Simplifies preprocessing of PCM measurement data

6. After determining the new statistics:
   identify almost not varying process model parameters
   - The calculated standard deviation of the process parameter will be very small

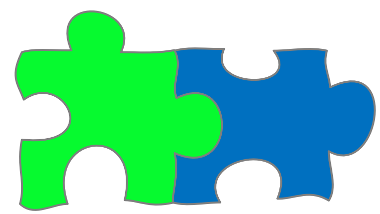
   ➔ Those model card parameter do not need to be varied in the spice models

Monte Carlo based validation of determined statistics

- Allows comparisons of new model statistics to target and original statistics
- Three PCM covariance matrixes to compare
  - PCM target covariance
  - Covariance based on newly created statistics (result of Monte Carlo analysis)
  - Covariance based on original spice models (result of Monte Carlo analysis)



newly created statistics



Target

- New result displays
  - Correlation matrix
    - multi-column/row sort
    - rearranging columns/rows manually
  - Delta in correlation matrix
  - Comparison of standard deviation

Comparison of the standard deviations

- NMOS and PMOS PCMs of an IC technology
- Standard deviation is matched very well

WiCkeD - Statistical Fit (Node 11) <1>

File Analysis Options Help

PCM sigma values | PCM sigma ranges | PCM correlations

PCM	PCM sigma target	PCM sigma simulated new
Vt_N_10/4_gm/5	22.22m	21.53m
G_N_10/4_gm/5	5.556u	5.635u
Vt_N_10/4_250n/0.1	22.22m	22.28m
Vt_N_10/4_25n/5	22.22m	22.2m
Vt_N_10/4_250n/5	22.22m	22.35m
Dibl_N_10/4_5	1.333m	114.9u
Slope_N_10/4_5	2m	378u
Ron_N_10/4_5/10u	55.56	8.426
Isat_N_10/4_5/5	44.44u	44.45u
Vt_P_10/3_gm/5	28.89m	21.54m
G_P_10/3_gm/5	6.667u	6.02u
Vt_P_10/3_133n/0.1	28.89m	22.41m
Vt_P_10/3_13.3n/5	28.89m	22.56m
Vt_P_10/3_133n/5	22.22m	22.49m
Dibl_P_10/3_5	1.778m	1.78m
Slope_P_10/3_5	1.556m	662.5u
Ron_P_10/3_5/10u	133.3	31.82
Isat_P_10/3_5/5	11.11u	28.33u

Spectre model file written to

Target PCM correlations

- NMOS and PMOS PCMs of an IC technology

WiCkeD - Statistical Fit (Node 9) <1>

File Analysis Options Help

PCM sigma values | PCM sigma ranges | PCM correlations

Target PCM correlation (TARGET)

Relevant NMOS PCMs: Vt_N_10/4_250n/5, Isat_N_10/4_5/5, G_N_10/4_gm/5, Ron_N_10/4_5/10u

Relevant PMOS PCMs: Vt_P_10/3_133n/5, Isat_P_10/3_5/5, G_P_10/3_gm/5, Ron_P_10/3_5/10u

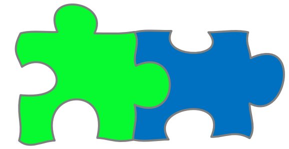
Further NMOS/PMOS PCMs: remaining rows

PCM	Vt_N_10/4_250n/5	Isat_N_10/4_5/5	G_N_10/4_gm/5	Ron_N_10/4_5/10u	Vt_P_10/3_133n/5	Isat_P_10/3_5/5	G_P_10/3_gm/5	Ron_P_10/3_5/10u	Vt_N_10/4_250n/0.1	Vt_P_10/3_gm/5	Vt_P_10/3_133n/0.1	Vt_P_10/3_13.3n/5	Vt_N_10/4_gm/5	Vt_N_10/4_25n/5	Slope_P_10/3_5	Slope_N_10/4_5	Dibl_P_10/3_5	Dibl_N_10/4_5
Vt_N_10/4_250n/5	+1.00	-0.91	-0.86	+0.76	-0.24	+0.54	-0.36	+0.55	+1.00	-0.28	-0.36	-0.33	+0.99	+1.00	-0.41	+0.79	-0.18	+0.13
Isat_N_10/4_5/5	-0.91	+1.00	+0.96	-0.94	+0.33	-0.72	+0.43	-0.73	-0.92	+0.36	+0.44	+0.41	-0.89	-0.91	+0.55	-0.78	+0.22	-0.15
G_N_10/4_gm/5	-0.86	+0.96	+1.00	-0.91	+0.33	-0.68	+0.43	-0.70	-0.87	+0.31	+0.40	+0.37	-0.82	-0.86	+0.51	-0.76	+0.22	-0.17
Ron_N_10/4_5/10u	+0.76	-0.94	-0.91	+1.00	-0.35	+0.76	-0.43	+0.78	+0.80	-0.38	-0.46	-0.42	+0.73	+0.80	-0.57	+0.69	-0.21	+0.11
Vt_P_10/3_133n/5	-0.24	+0.33	+0.33	-0.35	+1.00	-0.79	+0.09	-0.67	-0.35	+0.95	+1.00	+1.00	-0.23	-0.35	+0.46	-0.19	+0.01	-0.03
Isat_P_10/3_5/5	+0.54	-0.72	-0.68	+0.76	-0.79	+1.00	-0.38	+0.97	+0.54	-0.72	-0.80	-0.77	+0.53	+0.53	-0.68	+0.42	-0.13	+0.06
G_P_10/3_gm/5	-0.36	+0.43	+0.43	-0.43	+0.09	-0.38	+1.00	-0.44	-0.35	-0.18	+0.11	+0.06	-0.34	-0.35	+0.52	-0.24	+0.23	+0.04
Ron_P_10/3_5/10u	+0.55	-0.73	-0.70	+0.78	-0.67	+0.97	-0.44	+1.00	+0.54	-0.60	-0.67	-0.65	+0.53	+0.54	-0.65	+0.41	-0.13	+0.03
Vt_N_10/4_250n/0.1	+1.00	-0.92	-0.87	+0.80	-0.35	+0.54	-0.35	+0.54	+1.00	-0.28	-0.36	-0.33	+0.99	+1.00	-0.41	+0.78	-0.18	+0.19
Vt_P_10/3_gm/5	-0.28	+0.36	+0.31	-0.38	+0.95	-0.72	-0.18	-0.60	-0.28	+1.00	+0.95	+0.95	-0.28	-0.27	+0.37	-0.15	-0.03	-0.05
Vt_P_10/3_133n/0.1	-0.36	+0.44	+0.40	-0.46	+1.00	-0.80	+0.11	-0.67	-0.36	+0.95	+1.00	+1.00	-0.36	-0.36	+0.46	-0.20	+0.07	-0.03
Vt_P_10/3_13.3n/5	-0.33	+0.41	+0.37	-0.42	+1.00	-0.77	+0.06	-0.65	-0.33	+0.95	+1.00	+1.00	-0.33	-0.33	+0.41	-0.18	+0.00	-0.03
Vt_N_10/4_gm/5	+0.99	-0.89	-0.82	+0.73	-0.23	+0.53	-0.34	+0.53	+0.99	-0.28	-0.36	-0.33	+1.00	+0.99	-0.41	+0.79	-0.18	+0.14
Vt_N_10/4_25n/5	+1.00	-0.91	-0.86	+0.80	-0.35	+0.53	-0.35	+0.54	+1.00	-0.27	-0.36	-0.33	+0.99	+1.00	-0.41	+0.76	-0.18	+0.14
Slope_P_10/3_5	-0.41	+0.55	+0.51	-0.57	+0.46	-0.68	+0.52	-0.65	-0.41	+0.37	+0.46	+0.41	-0.41	-0.41	+1.00	-0.30	+0.07	-0.02
Slope_N_10/4_5	+0.79	-0.78	-0.76	+0.69	-0.19	+0.42	-0.24	+0.41	+0.78	-0.15	-0.20	-0.18	+0.79	+0.76	-0.30	+1.00	-0.17	-0.01
Dibl_P_10/3_5	-0.18	+0.22	+0.22	-0.21	+0.01	-0.13	+0.23	-0.13	-0.18	-0.03	+0.07	+0.00	-0.18	-0.18	+0.07	-0.17	+1.00	-0.03
Dibl_N_10/4_5	+0.13	-0.15	-0.17	+0.11	-0.03	+0.06	+0.04	+0.03	+0.19	-0.05	-0.03	-0.03	+0.14	+0.14	-0.02	-0.01	-0.03	+1.00

Spectre model file written to /spectre_statistics.scs.

Comparing the newly created model statistics with the target

- NMOS and PMOS PCMs of an IC technology
- Correlations match very well



WiCkeD - Statistical Fit (Node 9) <1>

File Analysis Options Help

PCM sigma values | PCM sigma ranges | PCM correlations

Difference between NEW and TARGET

PCM	Vt_N_10/4_250n/5	Isat_N_10/4_5/5	G_N_10/4_gm/5	Ron_N_10/4_5/10u	Vt_P_10/3_133n/5	Isat_P_10/3_5/5	G_P_10/3_gm/5	Ron_P_10/3_5/10u	Vt_N_10/4_250n/0.1	Vt_P_10/3_gm/5	Vt_P_10/3_133n/0.1	Vt_P_10/3_13.3n/5	Vt_N_10/4_gm/5	Vt_N_10/4_25n/5	Slope_P_10/3_5	Slope_N_10/4_5	Dibl_P_10/3_5	Dibl_N_10/4_5
Vt_N_10/4_250n/5	+0.00	-0.00	-0.00	+0.01	-0.05	-0.02	-0.01	+0.01	+0.00	+0.09	+0.06	+0.08	-0.02	-0.00	-0.13	-0.21	-0.00	-0.02
Isat_N_10/4_5/5	-0.00	-0.00	-0.00	-0.00	+0.04	+0.02	-0.00	-0.01	+0.00	-0.11	-0.06	-0.08	+0.02	+0.01	+0.04	+0.17	+0.00	-0.00
G_N_10/4_gm/5	-0.00	-0.00	-0.00	-0.00	+0.02	+0.02	-0.00	-0.01	+0.01	-0.08	-0.04	-0.06	-0.00	+0.02	+0.07	+0.18	+0.00	-0.00
Ron_N_10/4_5/10u	+0.01	-0.00	-0.00	-0.00	-0.03	-0.02	+0.01	+0.00	-0.03	+0.12	+0.06	+0.08	-0.01	-0.04	+0.05	-0.11	+0.00	+0.01
Vt_P_10/3_133n/5	-0.05	+0.04	+0.02	-0.03	-0.00	+0.00	-0.01	-0.00	+0.06	-0.01	-0.00	-0.00	-0.05	+0.06	-0.24	-0.06	+0.00	-0.00
Isat_P_10/3_5/5	-0.02	+0.02	+0.02	-0.02	+0.00	+0.00	-0.03	-0.00	-0.02	+0.08	+0.00	-0.01	-0.06	-0.04	+0.32	+0.12	-0.01	-0.05
G_P_10/3_gm/5	-0.01	-0.00	-0.00	+0.01	-0.01	-0.03	+0.00	+0.02	-0.01	-0.04	-0.01	+0.02	+0.06	+0.03	-0.51	-0.36	-0.00	-0.02
Ron_P_10/3_5/10u	+0.01	-0.01	-0.01	+0.00	-0.00	-0.00	+0.02	+0.00	+0.01	+0.07	-0.01	-0.01	-0.02	-0.00	+0.27	+0.11	+0.00	+0.03
Vt_N_10/4_250n/0.1	+0.00	+0.00	+0.01	-0.03	+0.06	-0.02	-0.01	+0.01	+0.00	+0.08	+0.06	+0.08	-0.01	+0.00	-0.16	-0.24	-0.00	-0.02
Vt_P_10/3_gm/5	+0.09	-0.11	-0.08	+0.12	-0.01	+0.08	-0.04	+0.07	+0.08	+0.00	-0.01	-0.02	+0.04	+0.07	-0.07	+0.16	-0.03	-0.13
Vt_P_10/3_133n/0.1	+0.06	-0.06	-0.04	+0.06	-0.00	+0.00	-0.01	-0.01	+0.06	-0.01	+0.00	-0.00	+0.07	+0.06	-0.25	-0.07	+0.02	-0.00
Vt_P_10/3_13.3n/5	+0.08	-0.08	-0.06	+0.08	-0.00	-0.01	+0.02	-0.01	+0.08	-0.02	-0.00	+0.00	+0.09	+0.08	-0.26	-0.07	+0.01	+0.05
Vt_N_10/4_gm/5	-0.02	+0.02	-0.00	-0.01	-0.05	-0.06	+0.06	-0.02	-0.01	+0.04	+0.07	+0.09	+0.00	-0.00	-0.25	-0.38	+0.02	+0.17
Vt_N_10/4_25n/5	-0.00	+0.01	+0.02	-0.04	+0.06	-0.04	+0.03	-0.00	+0.00	+0.07	+0.06	+0.08	-0.00	-0.00	-0.15	-0.26	+0.01	+0.03
Slope_P_10/3_5	-0.13	+0.04	+0.07	+0.05	-0.24	+0.32	-0.51	+0.27	-0.16	-0.07	-0.25	-0.26	-0.25	-0.15	+0.00	+0.21	-0.10	-0.64
Slope_N_10/4_5	-0.21	+0.17	+0.18	-0.11	-0.06	+0.12	-0.36	+0.11	-0.24	+0.16	-0.07	-0.07	-0.38	-0.26	+0.21	+0.00	-0.06	-0.52
Dibl_P_10/3_5	-0.00	+0.00	+0.00	+0.00	+0.00	-0.01	-0.00	+0.00	-0.00	-0.03	+0.02	+0.01	+0.02	+0.01	-0.10	-0.06	+0.00	-0.00
Dibl_N_10/4_5	-0.02	-0.00	-0.00	+0.01	-0.00	-0.05	-0.02	+0.03	-0.02	-0.13	-0.00	+0.05	+0.17	+0.03	-0.64	-0.52	-0.00	+0.00

Done.

Relevant NMOS PCMs

Relevant PMOS PCMs

Further NMOS/PMOS PCMs

Further advances in WiCkeD Statistical Fit

- Input data
  - Further input file formats supported
  - Comments in input files allowed
  - Automatic matching of PCM names of data sets and of spice simulation
- Log
  - Detailed log added
- While operating
  - Progress bar added
  - Improved status messages
  - Extended error handling
  - Improved handling of missing PCM values
- Export of results
  - Export of sigma values and covariance matrixes to Excel format
  - Spice include file with determined correlations

Conclusion

- Ultimate goal of statistical modeling to reproduce the correlations between measurements of PCMs can be achieved
- Ready to use WiCkeD Statistical Fit solution to determine a correlated statistics
- Removing of mismatch effects from PCM measurements
- Support of different structures of correlation matrixes
- Several input data checks
- Monte Carlo based Validation of results

WiCkeD

Circuit Design Optimization

Thank You !