Modeling The Bipolar Transistor For Bandgap Reference Simulations

Michael O. Peralta , Medtronic , December 8, 2010

- It is common to model bipolar transistors by curve fitting measured vs simulated data for "ic vs vbe", "beta vs vbe", etc.

- Bandgap circuit designs can still have significant differences between the actual measured circuit behavior and simulation.

- This usually happens because the measured vs simulated temperature behavior of ΔV_{BE} *vs T* and V_{BE} *vs T* is not directly optimized (curve fit).

- Here we will show how to include ΔV_{BE} *vs T* and V_{BE} *vs T* behaviors so that the bjt model properly emphasizes the temperature fit for bandgap circuits.

V_{BE} Difference At Different Collector Currents

The equations for the same physical bipolar transistor in low level current injection at two collector currents at different V_{BE} can be very accurately described as:

$$I_{C1} = I_S \cdot e^{V_{BE1}/V_T} \qquad 1\text{-}1$$

$$I_{C2} = I_S \cdot e^{V_{BE2}/V_T} \qquad 1\text{-}2$$

where

$$V_T = \frac{k \cdot T}{q} \qquad 1\text{-}3$$

Select I_{C2} and I_{C1} such that their ratio is exactly *m*

$$\frac{I_{C2}}{I_{C1}} = e^{(V_{BE2}-V_{BE1})/V_T} = m \qquad \text{"exactly"} \qquad 1\text{-}4$$

Define $$\Delta V_{BE} = V_{BE2} - V_{BE1} \qquad 1\text{-}5$$

Then using equations 1-3 and 1-4 we see that,

$$\Delta V_{BE} = V_{BE2} - V_{BE1} = V_T \ln(m) = \frac{kT}{q} \ln(m) \qquad 1\text{-}6$$

As an example we will select $I_{C1} = 1\ nA$ and $I_{C2} = 10\ nA$. Since we selected

$\frac{I_{C2}}{I_{C1}} = m$ we see that $$I_{C2} = m\, I_{C1} = 10\, I_{C1} \qquad 1\text{-}7$$

Using Linear Interpolation To Assure That m = 10 Exactly

- When measuring gummel (icvb) curves, V_{BE} is incremented in steps and I_C current is measured.

- This means that hitting I_{C1} and I_{C2} at exactly *1nA* and *10nA* is very unlikely.

- Hence we will use interpolation to find the V_{BE}'s that correspond to exactly *1nA* and *10nA*.

- Because of the exponential relationship between V_{BE} and I_C we will use values of $ln(I_C)$ and V_{BE} in our linear interpolation scheme.

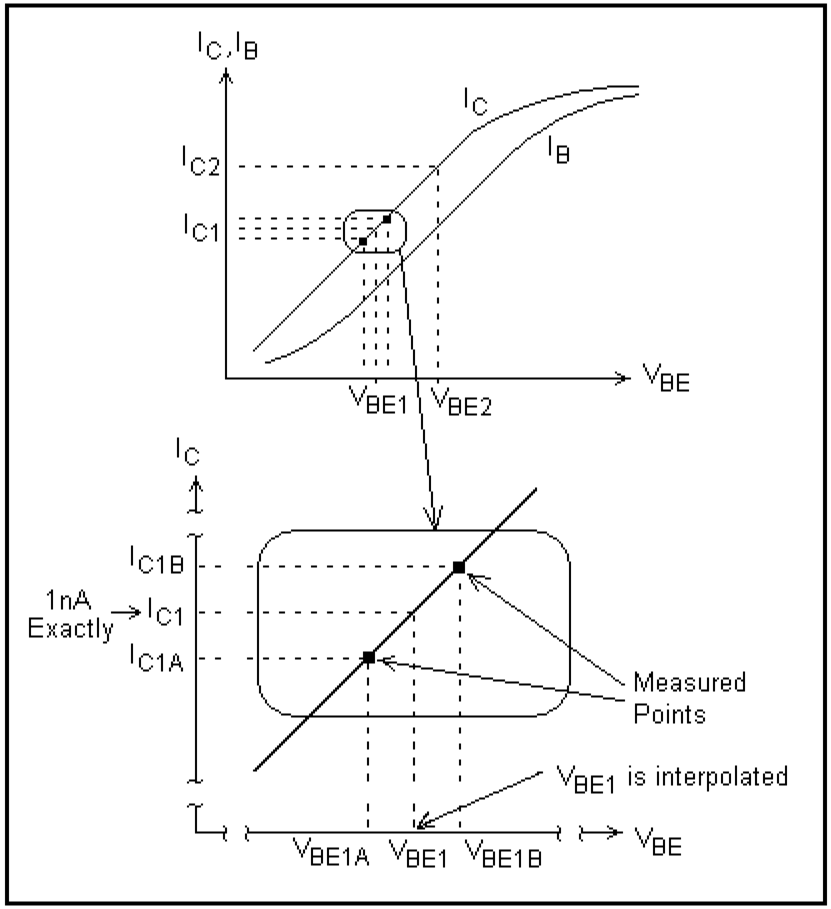

Figure 1. ln(I_C) , V_{BE} Interpolation

- As depicted in Figure 1 above, to determine V_{BE1} we find the I_{C1} just <u>below</u> *1nA* which we label I_{C1A} (the corresponding V_{BE} is labeled V_{BE1A}); and the I_{C1} <u>equal</u> to or <u>greater</u> than *1nA* is labeled I_{C1B} (the corresponding V_{BE} is labeled V_{BE1B}).

- From these pairs -- ($ln(I_{C1A}), V_{BE1A}$) and ($ln(I_{C1B}), V_{BE1B}$) -- we use linear interpolation to get V_{BE1} which corresponds to exactly $I_{C1} = 1nA$.

- For the $I_{C2} = 10nA$ case we use the same linear interpolation scheme to obtain the V_{BE2} value corresponding to exactly $I_{C2} = 10nA$.

Using equation 1-6 we then obtain

$$\Delta V_{BE} = V_{BE2}(at\ 10\ nA) - V_{BE1}(at\ 1\ nA) = \frac{kT}{q} \ln(m) \qquad 1\text{-}8$$

- Notice that both V_{BE1} and V_{BE2} are for the same transistor.

- Since the measurement sweep (depicted in Figure 1) is rapid and operated at low power (*1nA* and *10nA*), then V_{BE1} and V_{BE2} are practically at the same temperature, T.

- By contrast, in a bandgap circuit application, the different currents are created by two physically separate transistors -- which can be affected by device-to-device mismatch.

- At another temperature, T' , we keep <u>exactly</u> the same ratio between I'_{C2} and I'_{C1} as we did for temperature T -- that is $I_{C2} / I_{C1} = m$ (Eq. 1-4):

$$\frac{I'_{C2}}{I'_{C1}} = e^{\Delta V'_{BE}/V_T} = m \qquad \text{1-9}$$

$$\Delta V'_{BE} = V'_{BE2} - V'_{BE1} = V'_T \ln(m) = \frac{k\,T'}{q} \ln(m) \qquad \text{1-10}$$

- Again we use linear interpolation at the new temperature, T', to get V'_{BE1} and V'_{BE2} at <u>exactly</u> *1nA* and *10nA* respectively, thereby assuring that we have exactly m = 10.

Using Several Temperatures To Curve Fit Bandgap ΔV_{BE} *vs T* and V_{BE} *vs T*

- Say we take measurements at 4 different temperatures $T,\ T',\ T'',\ T'''$ then we would have 4 different $\Delta V_{BE}'s$ for measured and corresponding simulated $\Delta V_{BE}'s$:

Measured $\Delta V_{BE\_M},\ \Delta V_{BE\_M}',\ \Delta V_{BE\_M}'',\ \Delta V_{BE\_M}'''$ 1-11

Simulated $\Delta V_{BE\_S},\ \Delta V_{BE\_S}',\ \Delta V_{BE\_S}'',\ \Delta V_{BE\_S}'''$ 1-12

- At this point the simulated $\Delta V_{BE}'s$ are curve fit (optimized) to the corresponding measured $\Delta V_{BE}'s$. Figure 2 illustrates the curve fitting of the measured vs simulated $\Delta V_{BE}'s$. As seen through Eq. 1-8, the $\Delta V_{BE}'s$ are Proportional To Absolute Temperature (PTAT).

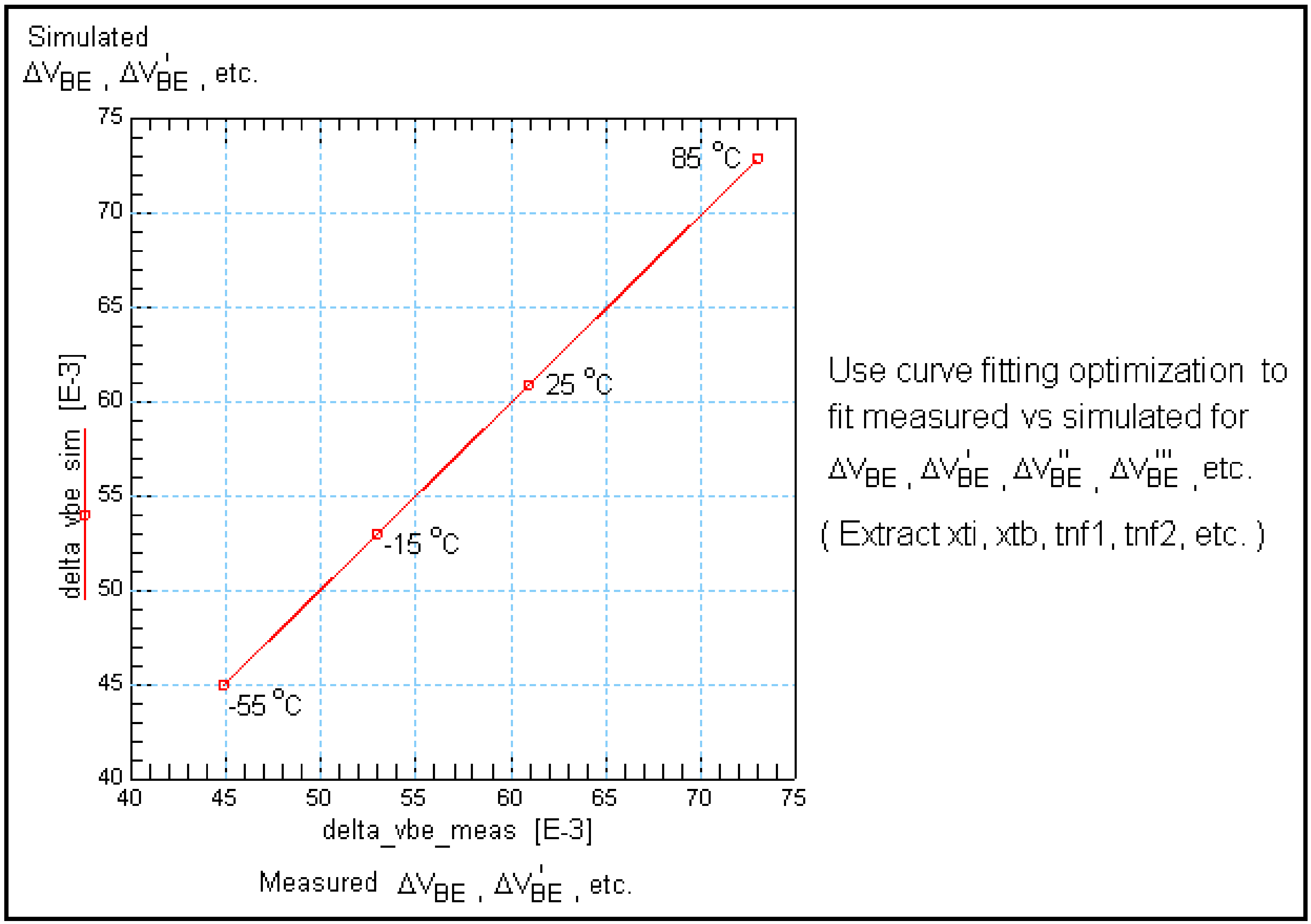

Figure 2. Curve Fit Measured vs Simulated ΔV_{BE}'s

- When we curve fit the measured vs simulated ΔV_{BE} curve we also curve fit the icvb (gummel curves), and the other measured vs simulated curves that are usually curve fit over temperature.

- Several curves are curve fit simultaneously by assigning different "weights" to each different curve set.

- Since the ΔV_{BE} measured vs simulated curve set has only 4 or 5 data points (temperatures) we weight that curve by a factor of 100 to 300 compared to the other typical curves (icvb =forward gummel curves, etc.)

- These other typical curves (icvb etc.) usually contain hundreds of data points and so they are given a weight of about 1.

- It is important to note that, in modern simulators (spectre, hspice, etc.), in addition to the "usual" temperature parameters (xti, xtb, etc.) there are several parameters that are also given temperature dependencies (usually first and second order). Refer to Appendix B.

- Among some of the parameters that are given temperature coefficients are: nf, nr, nc, ne, vaf, var, bf, br, ikf, ikr. This helps immensely in getting a good fit while keeping the parameters as close to physical as possible.

- As well as curve fitting the measured vs simulated $\Delta V_{BE}'s$ we also curve fit the measured vs simulated V_{BE}'s over the temperatures measured (V_{BE} vs T behavior). This is illustrated in Figure 3 below.

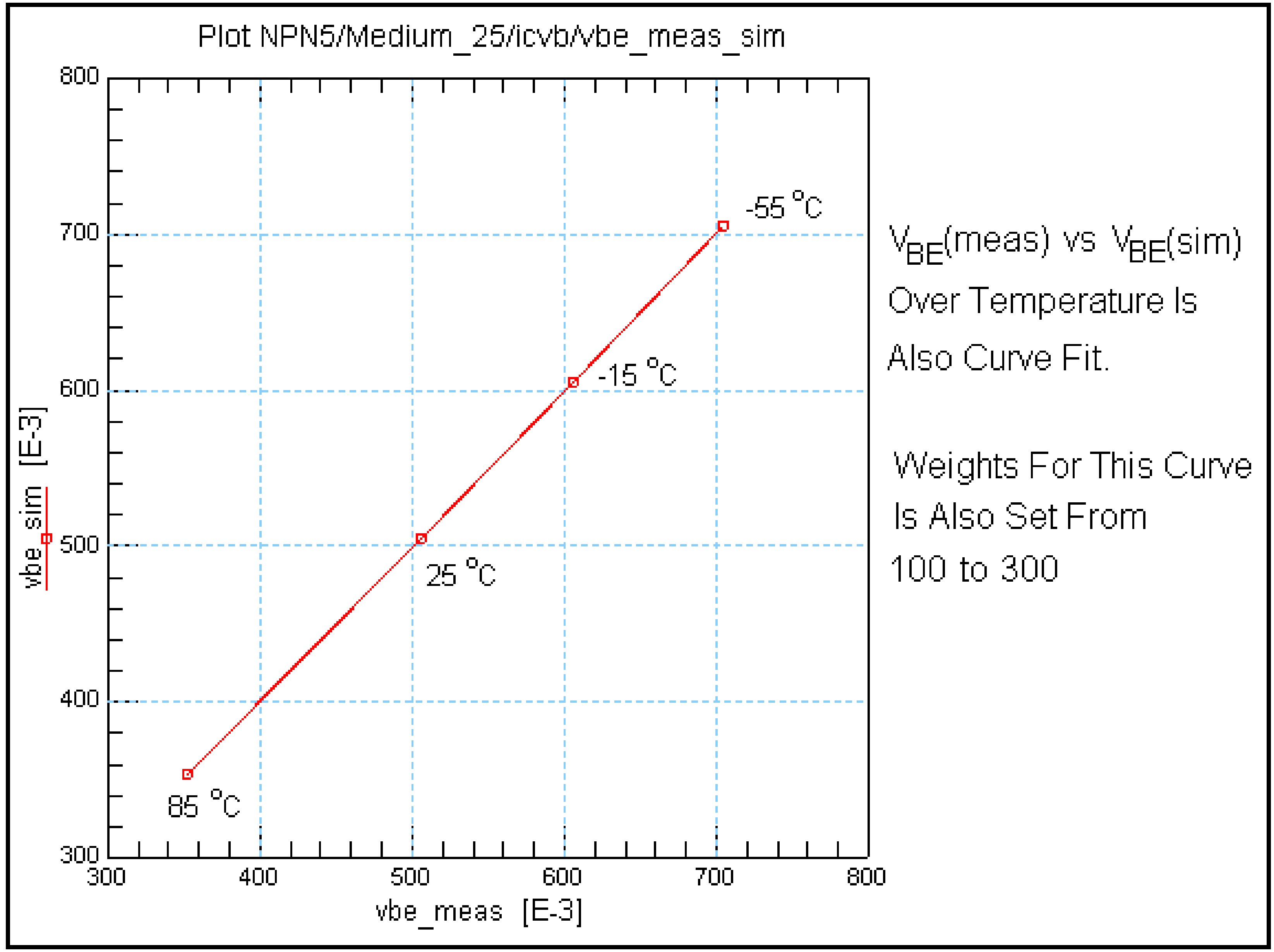

Figure 3. Curve Fit Measured vs Simulated V_{BE}'s

- Again, since we will only have 4 or 5 data points (temperatures) we also weight the V_{BE} measured vs simulated curve set by a factor of 100 to 300 compared to the other typical curves. Notice that this curve fit is also ***simultaneously*** performed with the ΔV_{BE} curve fit.

- Before performing the simultaneous ΔV_{BE} *vs T* and V_{BE} *vs T* curve fits, we perform curve fits at room temperature to extract ic vs vbe , beta vs vbe, Forward Early Voltage (V_{AF}), Reverse Early Voltage (V_{AR}), and others as deemed appropriate. This gives an extraction "baseline" for parameters that do not need to be co-extracted with the ΔV_{BE} *vs T* and V_{BE} *vs T* curve fitting.

- It also prepares the way so that the ΔV_{BE} *vs T* and V_{BE} *vs T* curve fits converge faster to the final extraction results.

Advantage of Bandgap Centric Curve Fit Approach

- The advantage of simultaneously curve fitting the ΔV_{BE} *vs T* and V_{BE} *vs T* curves with the typical (icvb, beta, etc.) curves, assures that the parameters that affect bandgap circuit behavior is emphasized. Without this approach, it is not likely that the model parameters that affect bandgap circuit behavior will be optimized for bandgap circuits.

- The advantage of the Bandgap Centric approach is illustrated by comparing Figures 4,5,6 (not bandgap centric) against Figures 7,8,9 (bandgap centric).

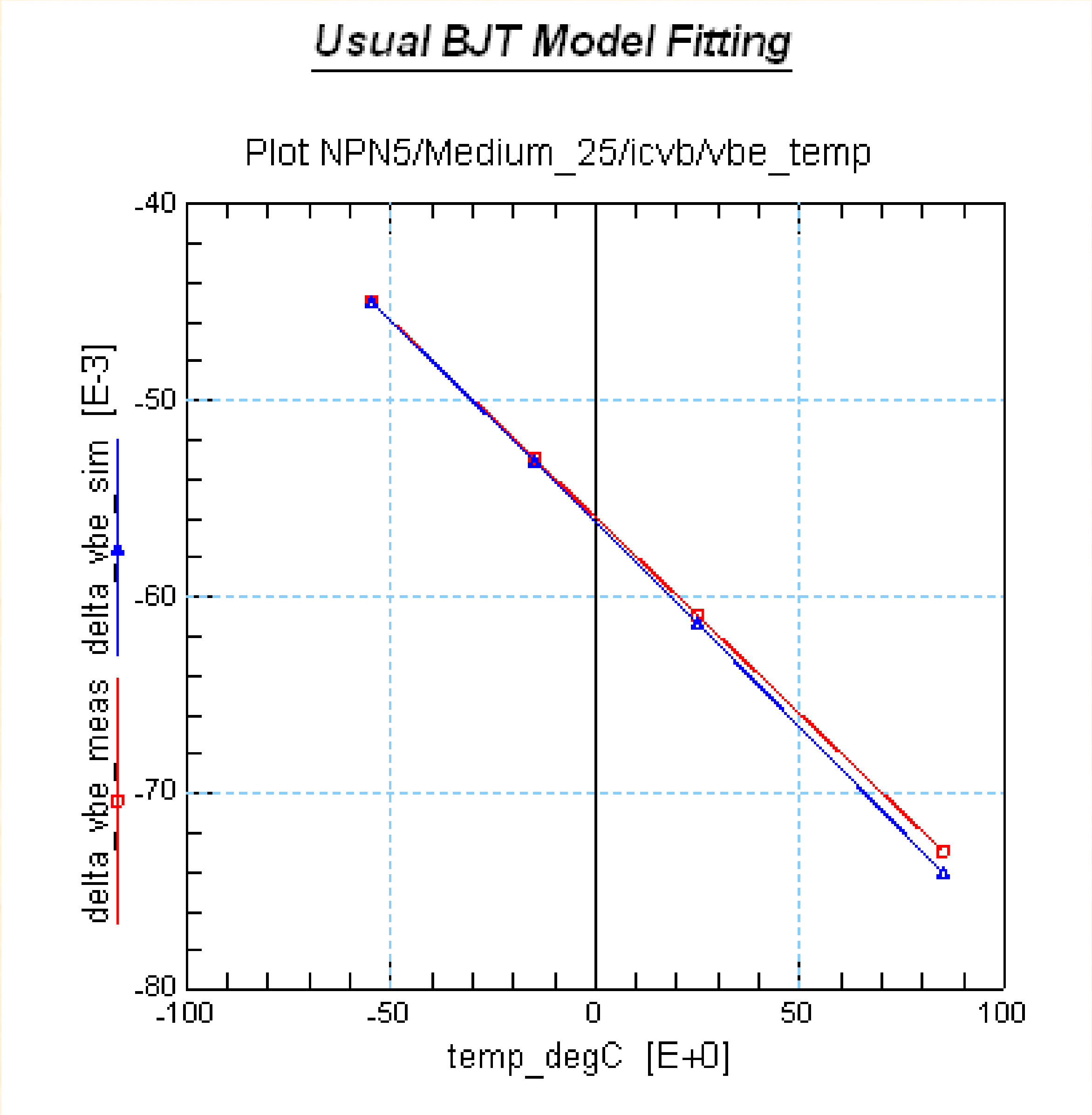

Figure 4. ΔV_{BE} Curve Fit Without Bandgap Centric Approach

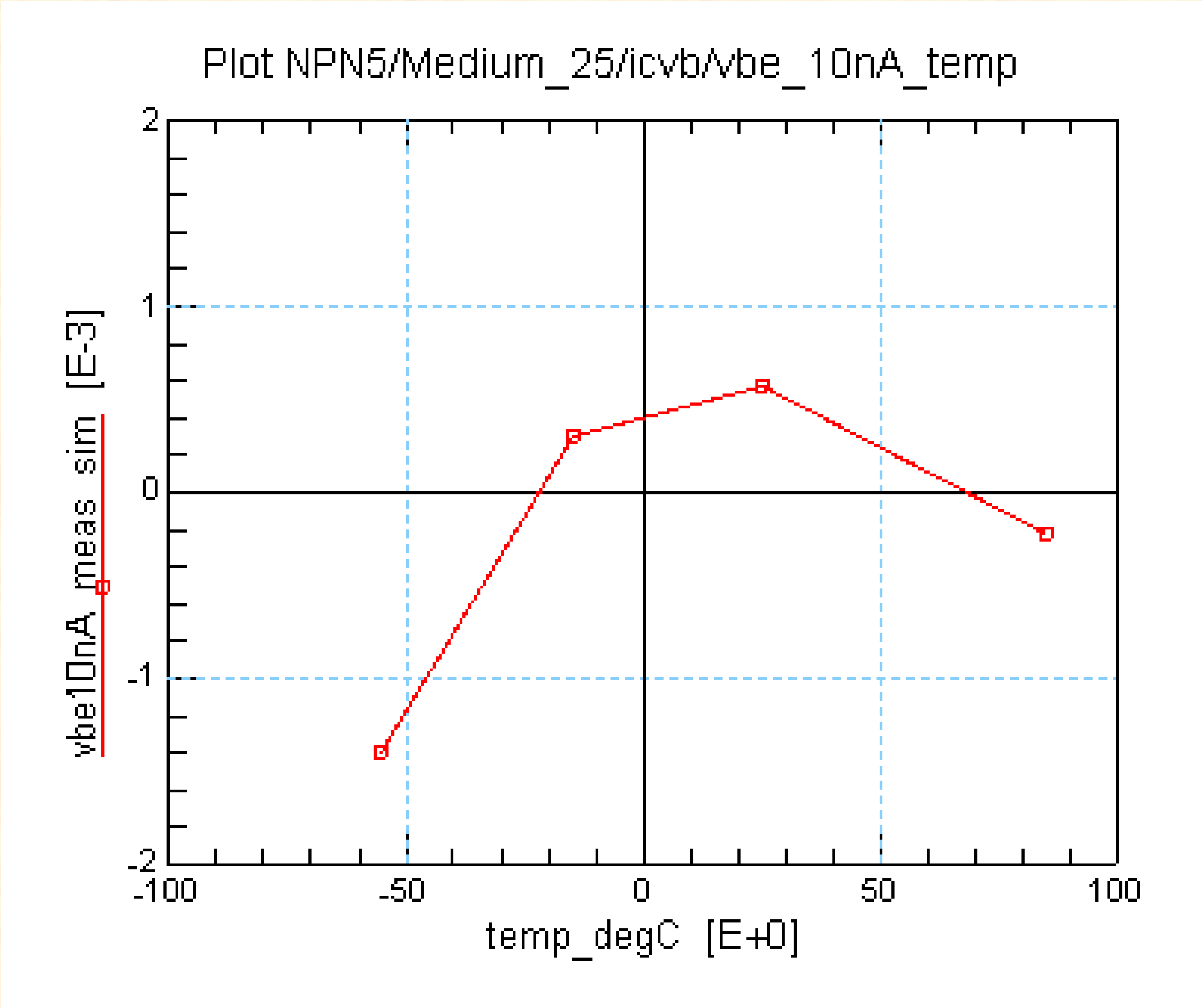

Figure 5. V_{BE}(meas) - V_{BE}(sim) Curve Fit Without Bandgap Centric Approach

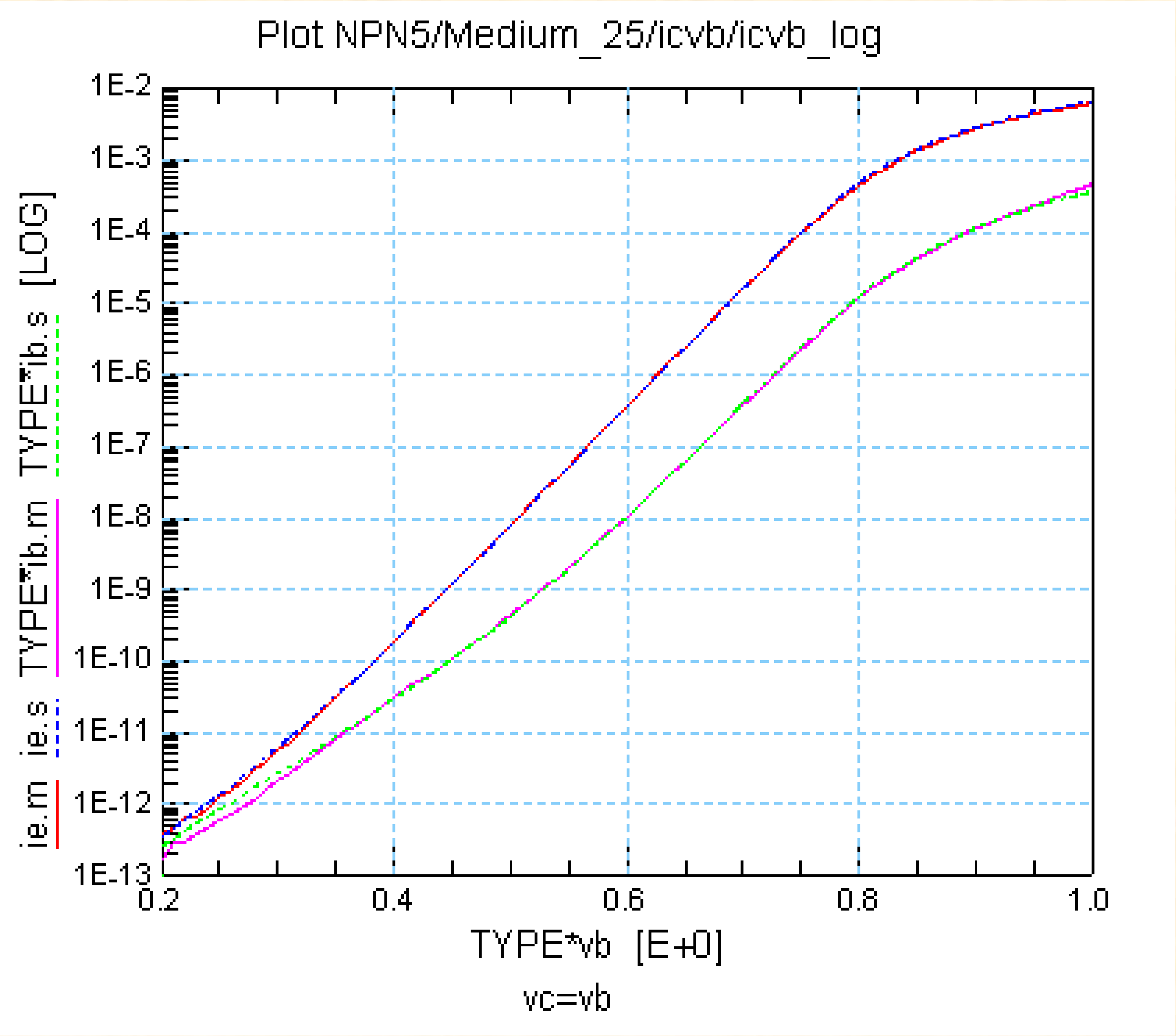

Figure 6. I_E vs vb Curve Fit <u>Without</u> Bandgap Centric Approach

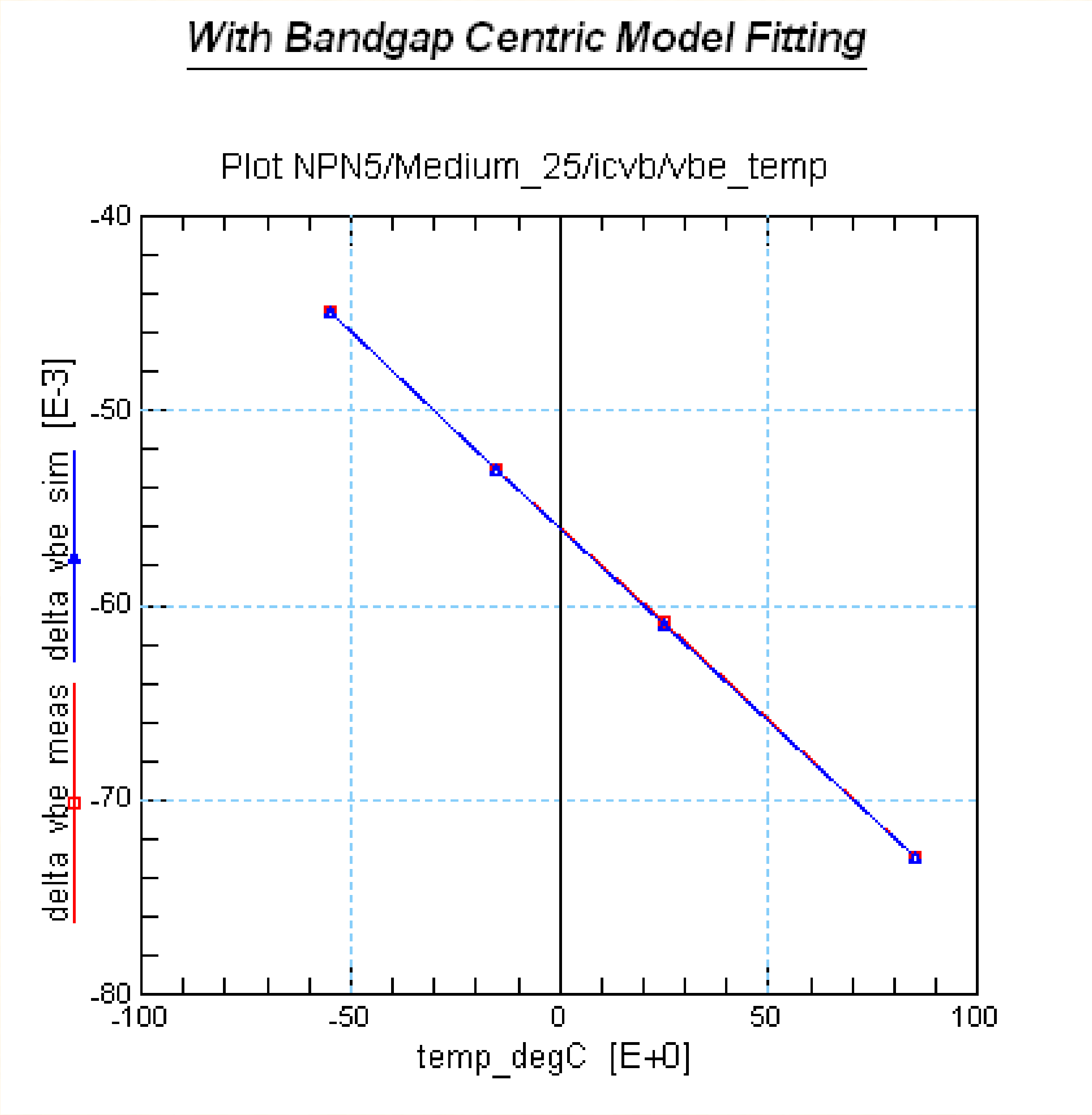

Figure 7. ΔV_{BE} Curve Fit With Bandgap Centric Approach

Compare this with Figure 4 which does not use the bandgap centric approach.

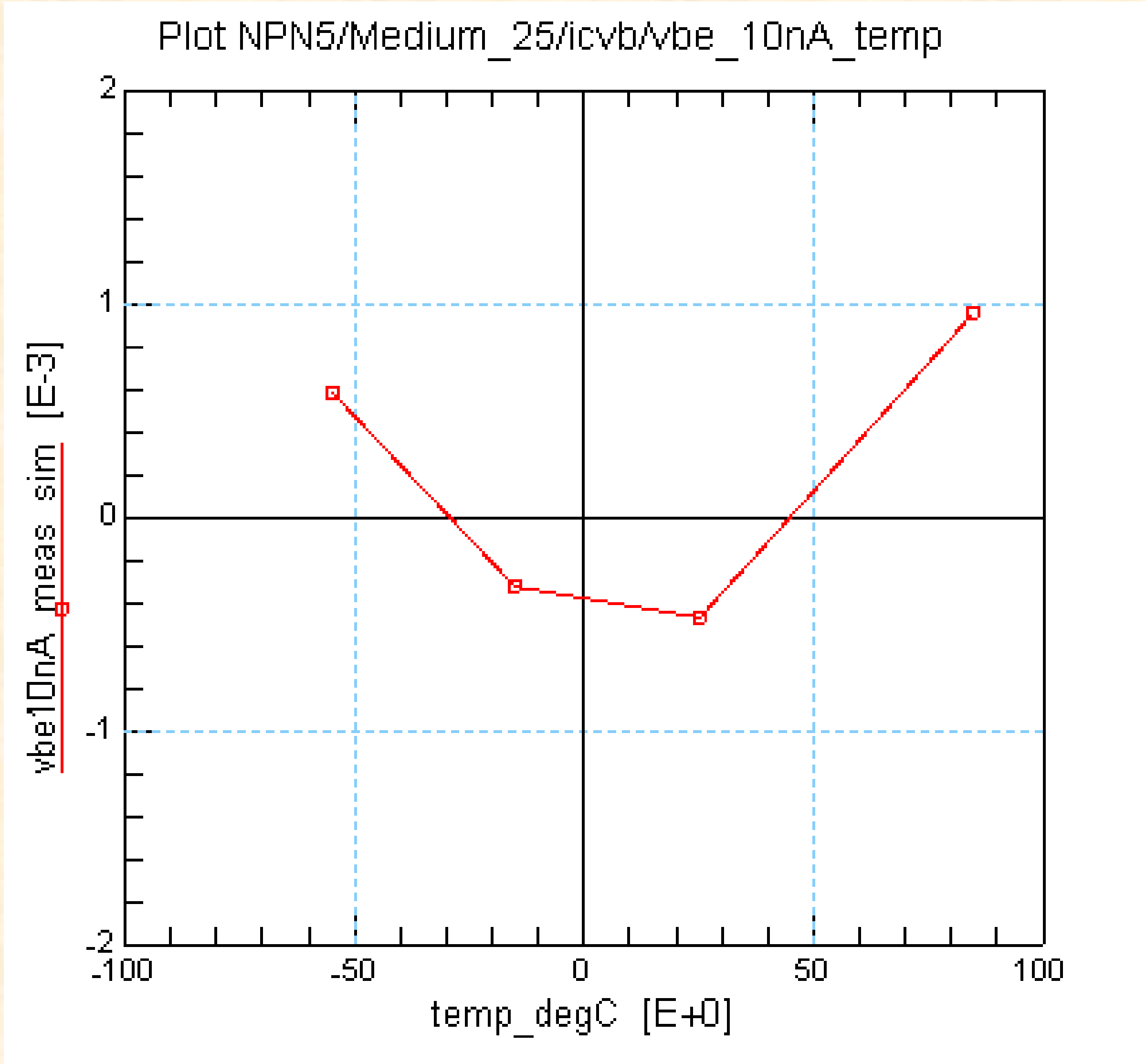

Figure 8. V_{BE}(meas) - V_{BE}(sim) Curve Fit With Bandgap Centric Approach

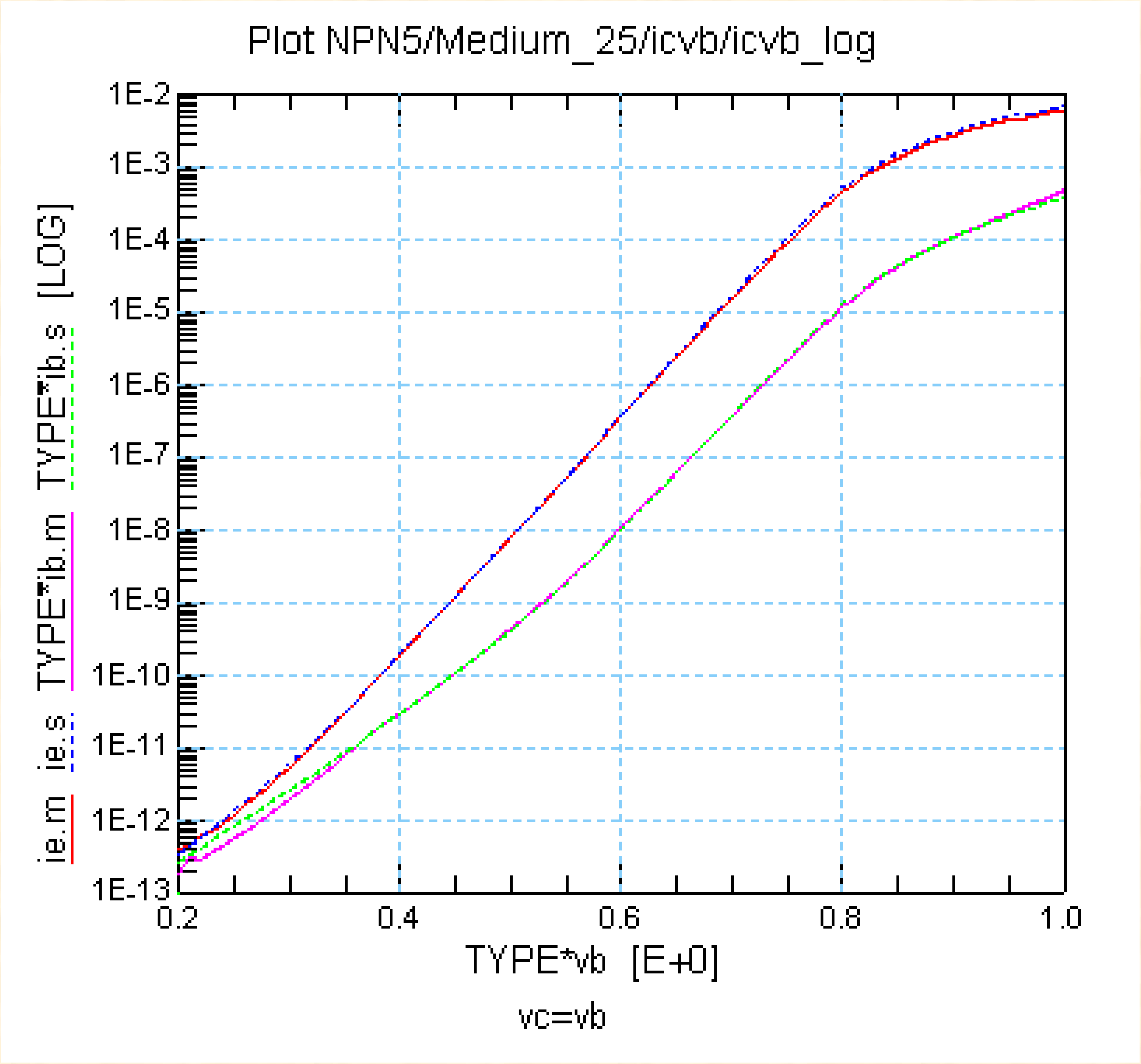

Figure 9. I_E vs vb Curve Fit With Bandgap Centric Approach

- It is important to notice that the I_E vs V_B curve fits (and the other typical modeling curves) may turn out good with or without the bandgap centric curve fit approach -- compare Figures 6 and 9.

- However, to get good curve fits for bandgap circuit applications we need good curve fits for the ΔV_{BE} *vs T* and V_{BE} *vs T* curves -- as we can see by comparing Figures 4 and 5 (not bandgap centric) against Figures 7 and 8 (bandgap centric).

Conclusion

- By directly including the ΔV_{BE} *vs T* and V_{BE} *vs T* temperature behaviors in the modeling curve fit procedures, parameters can be extracted so that bandgap circuit designs can be simulated with a higher degree of accuracy. This method does require that the modeling software used have the capability to properly "weight" the ΔV_{BE} *vs T* and V_{BE} *vs T* curves so these curves are emphasized enough in the midst of the curve fit optimizations.

- The accuracy of the method is enhanced by the fact that for each specific temperature, both the V_{BE1} and V_{BE2} used to determine each point of the ΔV_{BE} *vs T* and V_{BE} *vs T* curves, uses the same transistor. This avoids the usual device-to-device mismatch between the 1x and 8x (or other ratio) device sizes which are used in the usual bandgap circuit.

APPENDIX A: ΔV_{BE} vs. Temperature

Let us start with the Gummel-Poon equations (H. K. Gummel and H. C. Poon, "An integral charge control model of bipolar transistors," *Bell System Technical Journal,* vol. 49, pp. 827-52, 1970) :

$$I_C = \frac{I_S}{q_B} \cdot \left(e^{V_{BE}/n_F V_T} - 1\right) - \frac{I_S}{q_B} \cdot \left(e^{V_{BC}/n_R V_T} - 1\right) - \frac{I_S}{\beta_R} \cdot \left(e^{V_{BC}/n_R V_T} - 1\right) - I_{SC} \cdot \left(e^{V_{BC}/n_C V_T} - 1\right) \qquad \text{A-1}$$

$$I_{BE} = \frac{I_S}{\beta_F} \cdot \left(e^{V_{BE}/n_F V_T} - 1\right) + I_{SE} \cdot \left(e^{V_{BE}/n_E V_T} - 1\right) \qquad \text{A-2}$$

$$I_{BC} = \frac{I_S}{\beta_R} \cdot \left(e^{V_{BC}/n_R V_T} - 1\right) + I_{SC} \cdot \left(e^{V_{BC}/n_C V_T} - 1\right) \qquad \text{A-3}$$

$$q_B \cong q_1 = 1 + \frac{V_{BC}}{V_{AF}} + \frac{V_{BE}}{V_{AR}}$$ { low level injection } A-4

Let's force $V_{CB} = 0$ in which case we obtain,

$$I_C = \frac{I_S}{q_B} \cdot \left(e^{V_{BE} / n_F V_T} - 1 \right)$$ A-5

$$I_{BE} = \frac{I_S}{\beta_F} \cdot \left(e^{V_{BE} / n_F V_T} - 1 \right) + I_{SE} \cdot \left(e^{V_{BE} / n_E V_T} - 1 \right)$$ A-6

$$I_{BC} = 0$$ A-7

$$q_B \cong q_1 = 1 + \frac{V_{BE}}{V_{AR}}$$ A-8

For $V_{BE} >> V_T$ and letting $I_{C1} = 1nA$, $I_{C2} = 10nA = m\, I_{C1}$. Notice that I_{C1} and I_{C2} are for the same transistor. (In bandgap reference designs they are for matched transistors with different device areas.) With this we get for $I_{C1} = 1nA$ (with a bias of V_{BE1}),

$$I_{C1} = \frac{I_S}{q_{B1}} \cdot e^{V_{BE1} / n_F V_T} \qquad \text{A-9}$$

$$I_{BE1} = \frac{I_S}{\beta_F} \cdot e^{V_{BE1} / n_F V_T} + I_{SE} \cdot e^{V_{BE1} / n_E V_T} \qquad \text{A-10}$$

$$q_{B1} = 1 + \frac{V_{BE1}}{V_{AR}} \qquad \text{A-11}$$

For $I_{C2} = 10nA$ (with a bias of V_{BE2}) we see,

$$I_{C2} = \frac{I_S}{q_{B2}} \cdot e^{V_{BE2}/n_F V_T} \qquad \text{A-12}$$

$$I_{BE2} = \frac{I_S}{\beta_F} \cdot e^{V_{BE2}/n_F V_T} + I_{SE} \cdot e^{V_{BE2}/n_E V_T} \qquad \text{A-13}$$

$$q_{B2} = 1 + \frac{V_{BE2}}{V_{AR}} \qquad \text{A-14}$$

Taking the ratio I_{C2} / I_{C1} we obtain,

$$\frac{I_{C2}}{I_{C1}} = \frac{\left(I_S / q_{B2}\right) \cdot e^{V_{BE2}/n_F V_T}}{\left(I_S / q_{B1}\right) \cdot e^{V_{BE1}/n_F V_T}} = \frac{q_{B1}}{q_{B2}} \cdot e^{\left(V_{BE2} - V_{BE1}\right)/n_F V_T} \qquad \text{A-15}$$

Since we chose $I_{C1} = 1nA$ and $I_{C2} = 10nA = m\, I_{C1}$ we also have,

$$\frac{I_{C2}}{I_{C1}} = \frac{m \cdot I_{C1}}{I_{C1}} = m = \frac{q_{B1}}{q_{B2}} \cdot e^{(V_{BE2} - V_{BE1})/n_F V_T} = \frac{1 + V_{BE1}/V_{AR}}{1 + V_{BE2}/V_{AR}} \cdot e^{(V_{BE2} - V_{BE1})/n_F V_T} \qquad \text{A-16}$$

Rearranging we get,

$$m \cdot \left(\frac{1 + V_{BE2}/V_{AR}}{1 + V_{BE1}/V_{AR}} \right) = e^{(V_{BE2} - V_{BE1})/n_F V_T} \qquad \text{A-17}$$

Taking the natural log of both sides we obtain,

$$\ln\left[m \cdot \left(\frac{1 + V_{BE2}/V_{AR}}{1 + V_{BE1}/V_{AR}} \right) \right] = \frac{V_{BE2} - V_{BE1}}{n_F \cdot V_T} \qquad \text{A-18}$$

Solving this last expression for $V_{BE2} - V_{BE1}$ we get,

$$V_{BE2} - V_{BE1} = n_F\, V_T \cdot \left[\ln(m) + \ln\left(\frac{1 + V_{BE2}/V_{AR}}{1 + V_{BE1}/V_{AR}} \right) \right] \qquad \text{A-19}$$

With $V_{BE1} << V_{AR}$

$$V_{BE2} - V_{BE1} = n_F\, V_T \cdot \left[\ln(m) + \ln\left((1+V_{BE2}/V_{AR}) \cdot (1-V_{BE1}/V_{AR}) \right) \right] \qquad \text{A-20}$$

Expanding the expression,

$$(1+V_{BE2}/V_{AR}) \cdot (1-V_{BE1}/V_{AR}) = 1 + V_{BE2}/V_{AR} - V_{BE1}/V_{AR} - V_{BE1} \cdot V_{BE2}/V^2{}_{AR} \qquad \text{A-21}$$

and assuming $V_{BE1} << V_{AR}$ and $V_{BE2} << V_{AR}$ then we see that,

$$(1+V_{BE2}/V_{AR}) \cdot (1-V_{BE1}/V_{AR}) \cong 1 + V_{BE2}/V_{AR} - V_{BE1}/V_{AR} \qquad \text{A-22}$$

Defining x = V_{BE2} / V_{AR} - V_{BE1} / V_{AR} we see that |x| << 1 in which case

$$\ln(1+x) \cong x \qquad \text{A-23}$$

and so we have,

$$\ln\Big((1+V_{BE2}/V_{AR})\cdot(1-V_{BE1}/V_{AR})\Big) = V_{BE2}/V_{AR} - V_{BE1}/V_{AR} \qquad \text{A-24}$$

With this equation A-20 then simplifies to,

$$V_{BE2}-V_{BE1} = n_F\, V_T \cdot\Big[\ \ln(m) + V_{BE2}/V_{AR} - V_{BE1}/V_{AR}\ \Big] \qquad \text{A-25}$$

or collecting 1 / V_{AR} terms together we get,

$$V_{BE2}-V_{BE1} = n_F\, V_T \cdot\Big[\ \ln(m) + (V_{BE2}-V_{BE1})/V_{AR}\ \Big] \qquad \text{A-26}$$

Rearranging we get,

$$\left(V_{BE2} - V_{BE1}\right) - n_F\, V_T \cdot \left(V_{BE2} - V_{BE1}\right)/V_{AR} = n_F\, V_T \cdot \ln(m) \qquad \text{A-27}$$

or

$$\left(V_{BE2} - V_{BE1}\right) \cdot \left(1 - n_F\, V_T / V_{AR} \right) = n_F\, V_T \cdot \ln(m) \qquad \text{A-28}$$

Solving for V_{BE2} - V_{BE1} we obtain,

$$V_{BE2} - V_{BE1} = \frac{n_F \cdot V_T \cdot \ln(m)}{1 - n_F\, V_T / V_{AR}} \qquad \text{A-29}$$

Since $V_T << V_{AR}$ then

$$\frac{1}{1 - n_F\, V_T / V_{AR}} \cong 1 + \frac{n_F \cdot V_T}{V_{AR}} \qquad \text{A-30}$$

and so we find that,

$$V_{BE2} - V_{BE1} = n_F \cdot V_T \cdot \ln(m) \cdot \left(1 + \frac{n_F \cdot V_T}{V_{AR}}\right) \qquad \text{A-31}$$

Making the substitution for $V_T = k\,T / q$ we get,

$$\Delta V_{BE} = V_{BE2} - V_{BE1} = n_F \cdot \frac{k \cdot T}{q} \cdot \ln(m) \cdot \left(1 + \frac{n_F \cdot k \cdot T}{q \cdot V_{AR}}\right) \qquad \text{A-32}$$

Usually in a bandgap reference circuit analysis $\Delta V_{BE} = V_{BE2} - V_{BE1}$ is taken as,

$$\Delta V_{BE} = V_{BE2} - V_{BE1} = n_F \cdot \frac{k \cdot T}{q} \cdot \ln(m) \qquad \text{A-33}$$

But here, in equation A-32, we see a correction factor due to V_{AR}.

APPENDIX B: Some Temperature Expressions for the BJT

Saturation Current Temperature Dependence:

$$I_{SST} = I_{SS} \cdot \left(\frac{T}{T_{NOM}} \right)^{X_{TI} - X_{TB}} \cdot e^{\frac{E_g}{V_T}\left(\frac{T}{T_{NOM}} - 1 \right)} \qquad \text{B-1}$$

Similar expressions exist for I_{SE}, and I_{SC}.

Forward Beta Temperature Dependence:

$$B_{FT} = B_F \cdot \left(\frac{T}{T_{NOM}} \right)^{X_{TB}} \qquad \text{B-2}$$

A similar expression exists for B_R.

Forward Ideality Factor Temperature Dependence:

$$n_{ft} = n_f \cdot \left\{ 1 + t_{nf1}\left(T - T_{NOM}\right) + t_{nf2}\left(T - T_{NOM}\right)^2 \right\} \qquad \text{B-3}$$

Similar expressions exist for nr , nc , ne , vaf , var , ikf , ikr and others.

APPENDIX C: Expressions for PTAT Voltage Generation

Generation of PTAT (Proportional To Absolute Temperature) Voltage:

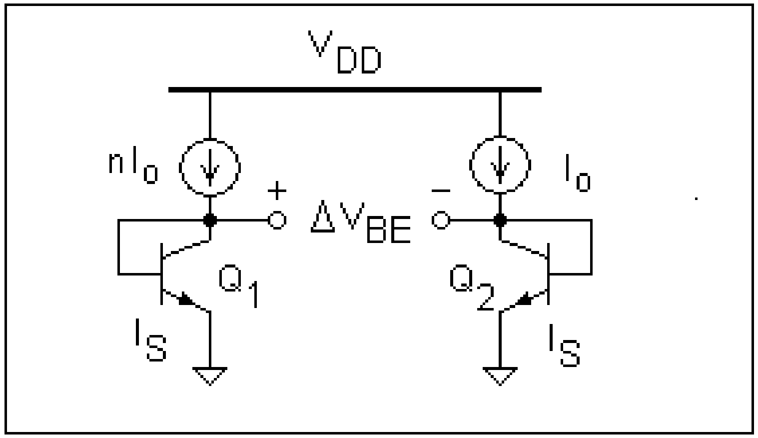

Figure C1: Generation of PTAT Voltage

If two matched transistors, $I_{S1} = I_{S2} = I_S$, are biased at collector currents of $n\,I_O$ and I_O (with negligible base currents), as shown in Figure C1, then

$$\Delta V_{BE} = V_{BE2} - V_{BE1} = V_T \cdot \ln\left(\frac{n \cdot I_O}{I_{S1}}\right) - V_T \cdot \ln\left(\frac{I_O}{I_{S2}}\right) \qquad \text{C-1}$$

$$\Delta V_{BE} = V_T \cdot \ln(n) = \frac{kT}{q} \cdot \ln(n) \qquad \text{C-2}$$

Generation of PTAT Voltage Involving Several or Different Area Transistors:

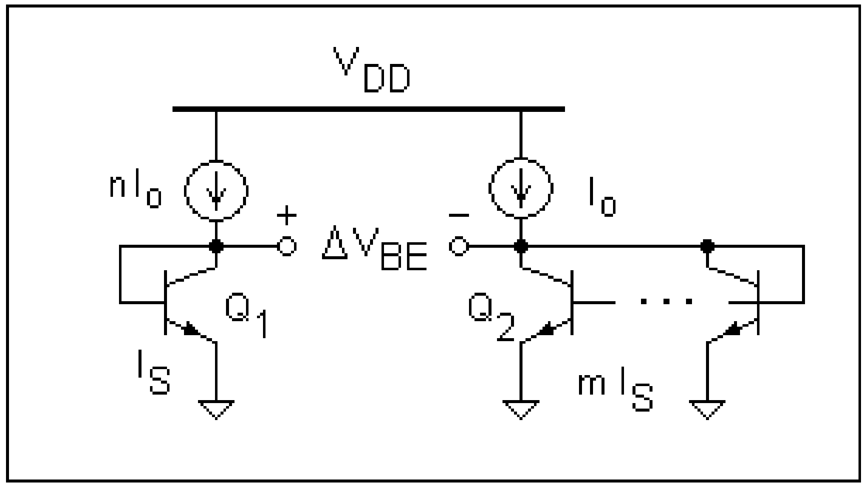

Figure C2:
Generation of PTAT Voltage Involving Multiple or Different Area Transistors

If the transistors are of different areas or multiples, as shown in Figure C2, then

$$\Delta V_{BE} = V_{BE2} - V_{BE1} = V_T \cdot \ln\left(\frac{n \cdot I_O}{I_S}\right) - V_T \cdot \ln\left(\frac{I_O}{m \cdot I_S}\right) \qquad \text{C-3}$$

$$\Delta V_{BE} = V_T \cdot \ln(n \cdot m) = \frac{kT}{q} \cdot \ln(n \cdot m) \qquad \text{C-4}$$