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9-12 SEPTEMBER
ESSERC 2024

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A review of charge-based MOS Transistor modeling

an Engineering and Educational Tool

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Outline

- ☐ The EKV3 charge-based model of the MOS transistor is a highly versatile tool:
  - A **full compact model** for circuit simulation
  - Used in commercial simulators and PDKs
  - Available in open-source tools such as NGSpice, QUCS
  - **OPEN SOURCE:** https://github.com/MatBucher/ekv3model
- ☐ The charge-based model: model quantities are related to circuit-level parameters
  - An **engineering tool** for design and education
  - COVERS WEAK-MODERATE-STRONG INVERSION seamlessly
  - Parameter extraction techniques are fully consistent with the model
- ☐ The model clarifies relationships among key circuit-design Figures-of-merit (FoMs) such as:
  - Transconductance,
  - Transconductance-to-current ratio,
  - Capacitance,
  - Transit frequency

Charge-based MOSFET model for $\mathrm{I_D}$

Drain current in all regions of MOSFET operation:

$$\mathrm{I_D} = \mathrm{I_0}\frac{\mathrm{W}}{\mathrm{L}} \cdot \left[i_f - i_r\right] = \mathrm{I_0}\frac{\mathrm{W}}{\mathrm{L}} \cdot \left[(q_s^2 + q_s) - (q_s^2 + q_d)\right]$$

Relationship among voltages and inversion charge $\mathrm{q_{S(D)}}$:

$$\frac{\mathrm{V_P} - \mathrm{V_{S(D)}}}{\mathrm{U_T}} = 2q_{s(d)} + ln\left(q_{s(d)}\right)$$

Inverse relationship given by Lambert $\mathrm{W_0}$ function:

$$q_{s(d)} = \frac{1}{2}\mathrm{W_0}[2 \cdot \exp(\frac{\mathrm{V_P} - \mathrm{V_{S(D)}}}{\mathrm{U_T}})]$$

Single equation for all operating regions
(**weak-moderate-strong inversion**)
Only parameters: $\mathrm{V_{TO}}$, n, $\mathrm{I_0}$

$$\mathrm{i_{f(r)}} = q_{s(d)}^2 + q_{s(d)}$$

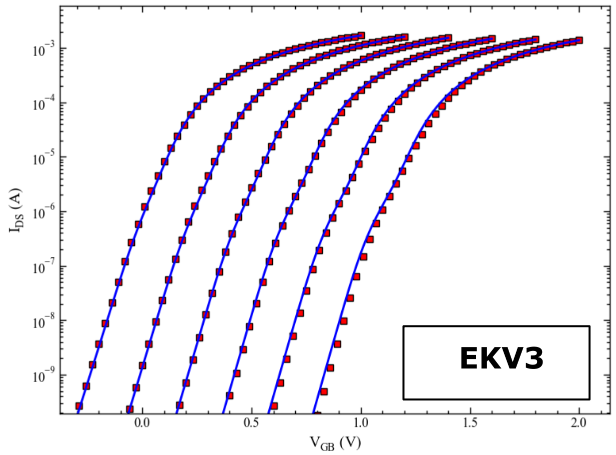
$$\Longleftrightarrow \quad q_{s(d)} = \mathrm{sqrt}\left[\frac{1}{4} + \mathrm{if_{(r)}}\right] - \frac{1}{2}$$

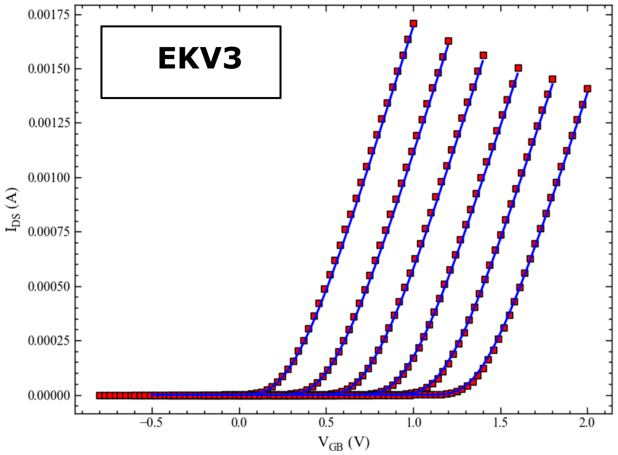
Technology Current: $\mathrm{I_0} = 2 \cdot n \cdot U_T^2 \cdot \mu \cdot C'_{ox}$

$$\mathrm{U_T} = \frac{\mathrm{kT}}{\mathrm{q}}$$

Pinch-off Voltage: $\mathrm{V_P} \cong \dfrac{\mathrm{V_G} - \mathrm{V_{TO}}}{\mathrm{n}}$

Slope Factor: $n \cong 1 + \dfrac{\gamma}{2 \cdot sqrt(2\Phi_\mathrm{F})}$





Transconductance $\mathbf{G_m}$ and $\mathbf{G_m/I_D}$ (long-channel)

Gate transconductance:

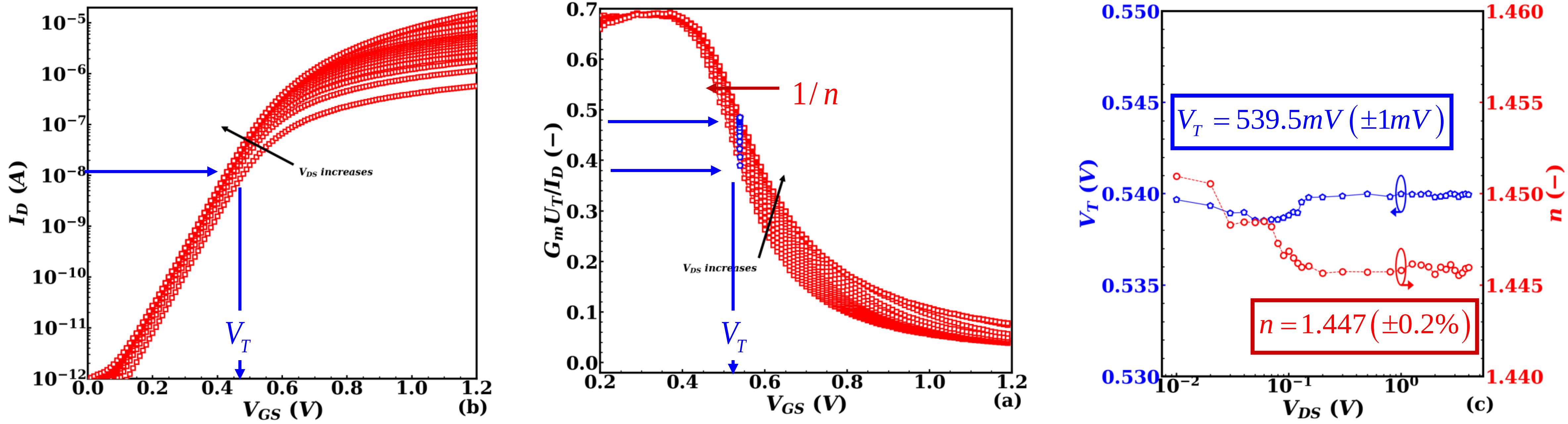
$$G_m \equiv \left.\frac{\partial I_D}{\partial V_G}\right|_{V_S,V_D} = \frac{I_{SPEC}}{U_T} \cdot \frac{q_S - q_D}{n}$$

Transconductance-to-current ratio:

$$\frac{G_m U_T}{I_D} = \frac{1}{n \cdot \left(1 + q_S + q_D\right)}$$

- ☐ Single equation for all operating regions (weak-moderate-strong inversion)!
- ☐ Only parameters: V_{TO}, n, I_0

V_{TO} and **n** extraction from G_M/I_D



Long-channel, thick-oxide NMOST: T_{ox}=20nm, W/L = 20um/20um, V_{DS} = 10mV … 4V.

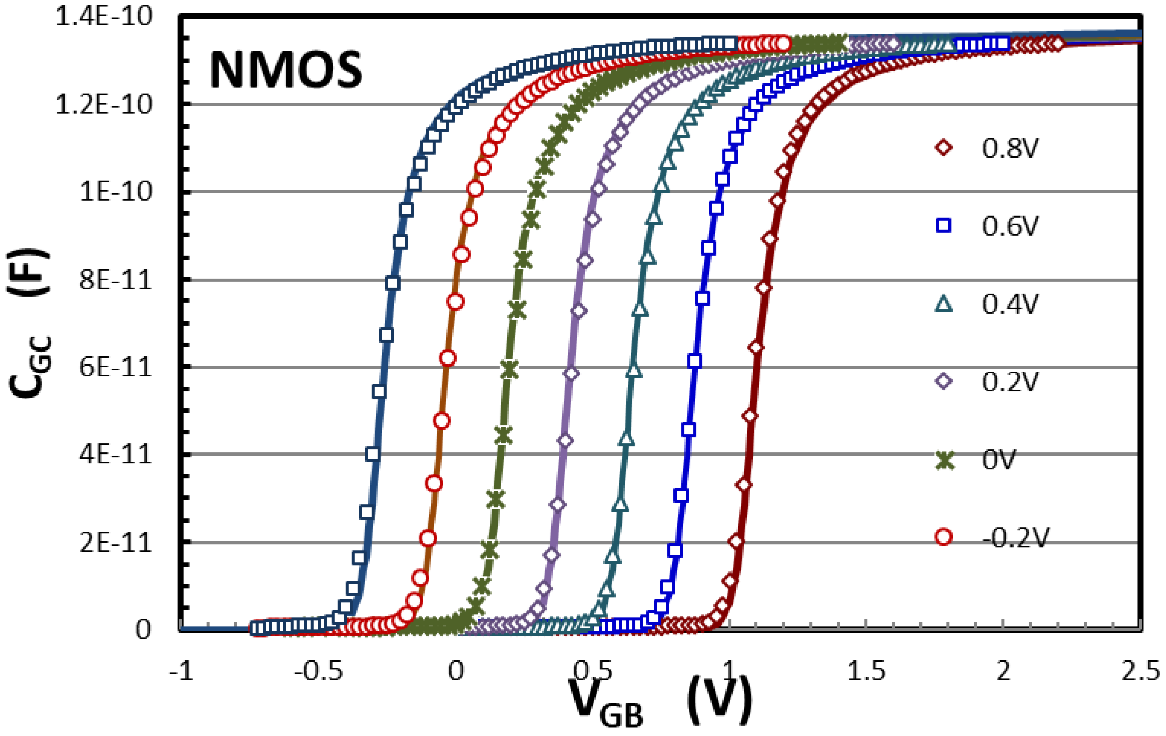
Threshold condition:

Slope factor n obtained from: $n = \left[\left.\frac{G_m U_T}{I_D}\right|_{\max,WI}\right]^{-1}$

$$V_P = V_S \Leftrightarrow \left.\frac{G_m U_T}{I_D}\right|_{V_P=V_S} = \frac{1}{n\cdot\left(1+\left.q_S\right|_{V_P=V_S}+\left.q_D\right|_{V_P=V_S}\right)} \Leftrightarrow \frac{1}{n\cdot\left(1.426+0.5\cdot W_0\left[2\exp\left(\left(-V_{DS}\right)/U_T\right)\right]\right)}$$

[REF] M. Bucher et al. IEEE LAEDC, 2023 10.1109/LAEDC58183.2023.10209129

Capacitance $\mathbf{C_{GC}}$ vs. mobile charge $\mathbf{q_s}$



$$C_{gc} = C_{gd} + C_{gs} = \frac{\partial Q_g}{\partial V_d} + \frac{\partial Q_g}{\partial V_s} = C_{OX}\frac{2\left(q_s^2 + q_d^2\right) + 8q_sq_d + 3(q_s + q_d)}{3\cdot(q_s + q_d + 1)^2}$$

In **linear mode** (V_D=V_S, q_d=q_s):

$$\boxed{C_{gc}\Big|_{q_d=q_s} = C_{OX}\frac{2q_s}{2q_s + 1}}$$

$$C_{OX} = C'_{ox}\cdot \mathrm{W}\cdot \mathrm{L} \quad where \quad C'_{ox} = \frac{\epsilon_{ox}}{T_{ox}}$$

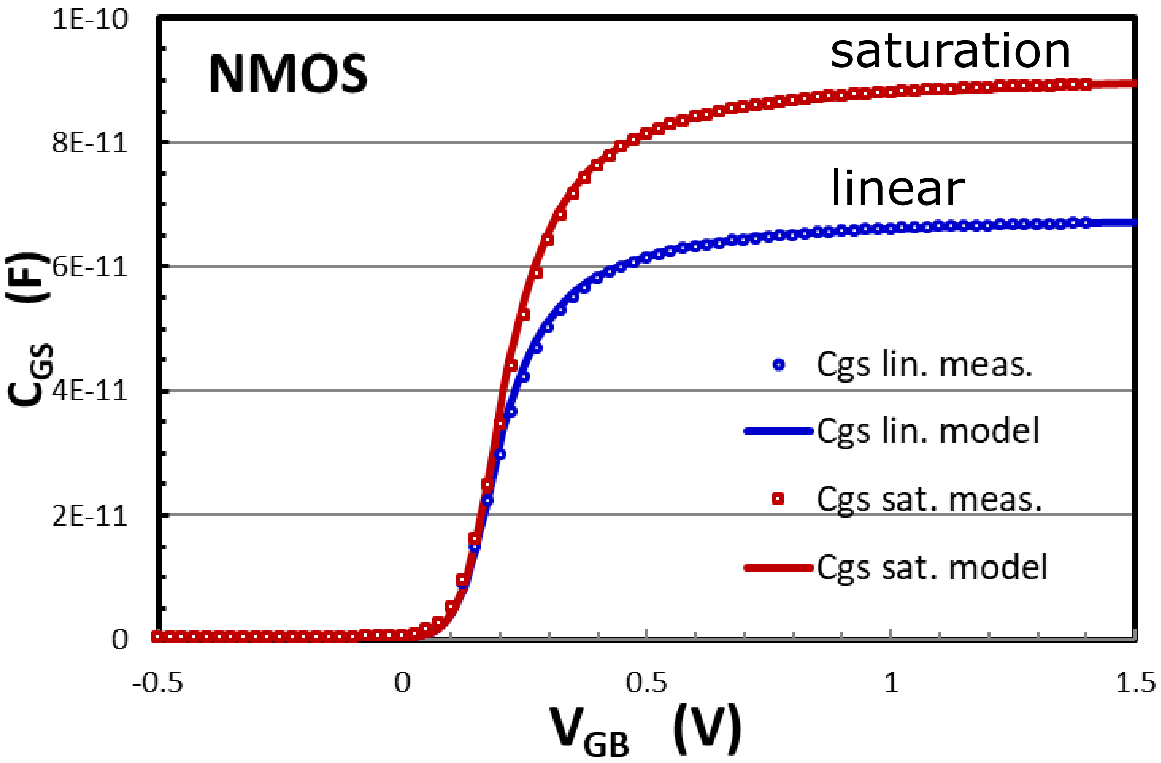
$$\frac{\mathrm{V_P} - \mathrm{V_{S(D)}}}{\mathrm{U_T}} = 2q_{s(d)} + ln\left(q_{s(d)}\right) \quad \text{where} \quad \mathrm{V_P} = \frac{\mathrm{V_G} - \mathrm{V_{TO}}}{\mathrm{n}}$$

$$\boxed{\mathrm{V_G} = \mathrm{V_{TO}} + n[V_S + \mathrm{U_T}(2q_s + ln(q_s))]}$$

- Gate-channel capacitance **measured** from split-CV in **linear mode** (V_{DS}=0V)
- Only parameters: C'_{ox}, V_{TO}, n
  - NMOS, 55nm bulk CMOS technology

V_{TO}	0.2	V
n	1.33	-
C_{ox}	0.0139	F/m^2

Capacitance $\mathbf{C_{GS}}$ vs. mobile charge $\mathbf{q_s}$



1. q_s is determined from measured C_{gc} in linear mode:

$$q_s = \frac{1}{2} \cdot \frac{c_{gc}}{1 - c_{gc}} \qquad c_{gc} = \frac{C_{gc}}{C_{OX}}$$

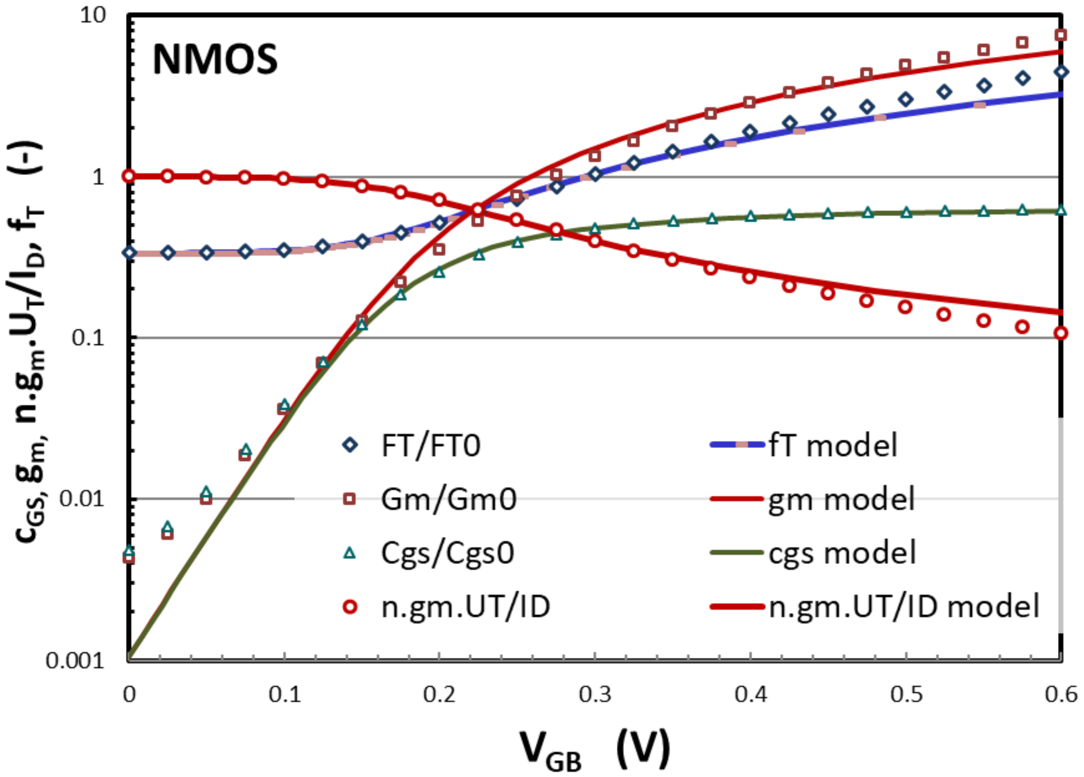
2. Now, C_{gs} is calculated in **saturation mode** ($V_D > V_P$, i.e. $q_d \ll q_s$):

$$C_{gs}\Big|_{sat.} = C_{OX} \frac{q_s}{3} \frac{2q_s + 3}{(q_s + 1)^2}$$

$$\mathrm{V_G} = \mathrm{V_{TO}} + n[V_S + \mathrm{U_T}(2q_s + ln(q_s))]$$

- ❑ Gate-source capacitance C_{gs} in **saturation mode** ($q_s \gg q_d$)
- ❑ Saturation-mode C_{gs} is obtained from measured C_{gs} in linear mode (via q_s)
- ❑ Same device, **same** model parameters: C'_{ox}, V_{TO}, n

F_T as a function of mobile charge $\mathbf{q_s}$ (sat.)



Combining G_m and C_{GS} models, F_T may be expressed as:

$$F_T = \frac{G_M}{2\pi C_{GS}} = \frac{\frac{I_0}{nU_T}\frac{W}{L}q_s}{2\pi \cdot C'_{ox} \cdot W \cdot L\,\frac{q_s}{3}\,\frac{2q_s+3}{(q_s+1)^2}}$$

$$\boxed{F_T = \frac{3 \cdot \mu}{\pi \cdot L^2} \cdot \frac{(q_s+1)^2}{2q_s+3}}$$

$$\frac{F_T}{F_{T0}} = \frac{(q_s+1)^2}{2q_s+3} \qquad F_{T0} = \frac{3 \cdot \mu}{\pi \cdot L^2}$$

- ❑ Analytical models coincide with “inferred measured” (via q_s) quantities.
- ❑ **Same** model parameters: C'_{ox}, V_{TO}, n

Conclusion

- ☐ The EKV3 charge-based model of the MOS transistor is a highly versatile tool:
  - A **full compact model** for circuit simulation
  - **OPEN SOURCE:** https://github.com/MatBucher/ekv3model
  - EKV3 to be evaluated for Open-Source PDK (IHP 130nm)
- ☐ EKV3 is seconded by engineering tool (the charge-based model) for design and education:
  - The model clarifies relationships among key circuit-design Figures-of-merit (FoMs) such as:
    - Transconductance (G_M),
    - Transconductance-to-current ratio (G_M/I_D),
    - Capacitance (C_{GS}),
    - Gain-bandwidth (GBW)
  - All these quantities are directly related to the **mobile charge** in the MOS channel.
- ☐ Analytical equations and a minimal parameter sets. Simple parameter extraction
- ☐ The model is a **highly valuable tool in educating analog design engineers**