Compact Modeling for Transistors Accommodating Inner Thermal Feedback Behavior

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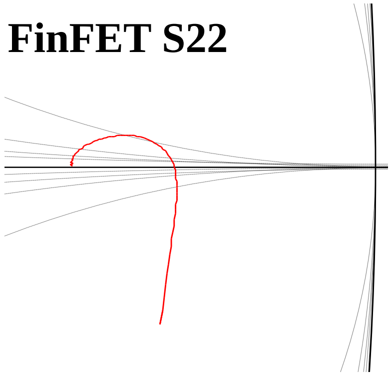
Context

- ☞ ☐ Background
- ☐ Theory and Derivation
- ☐ Model Development
  - ➢ FinFET Device Modeling
- ☐ Extended Application
- ☐ Summary

Self-heating Effect in Transistors

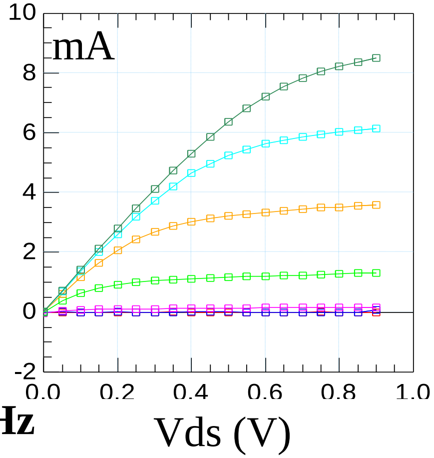
- ☐ **DC Behaviors: device temperature rise, reduction in on-current…..**
- ☐ **Frequency Behaviors: long signal delays, oscillation, inductive in-/output-impedance….**

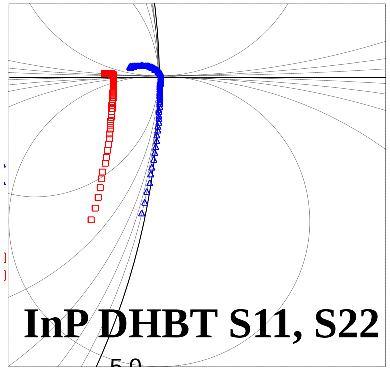
FinFET S22



Freq: 100KHz to 3.1GHz

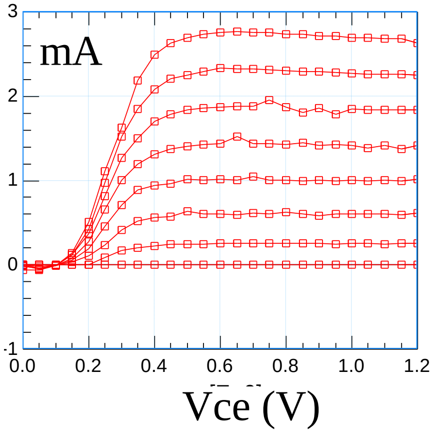
FinFET DC IV

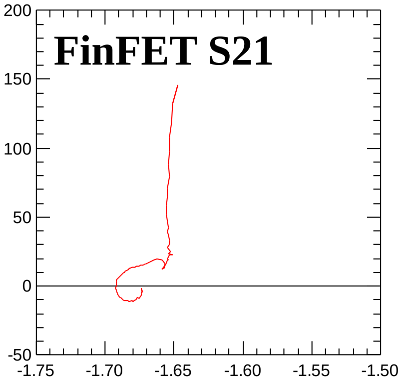




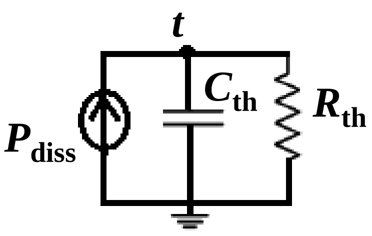
InP DHBT S11, S22

InP DHBT DC IV

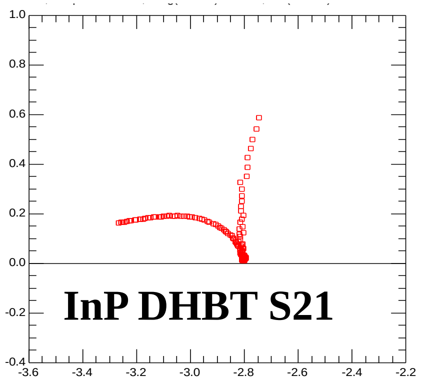




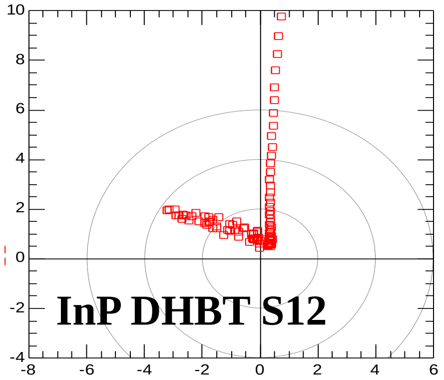
FinFET S21



Commonly Used Thermal Sub-circuit



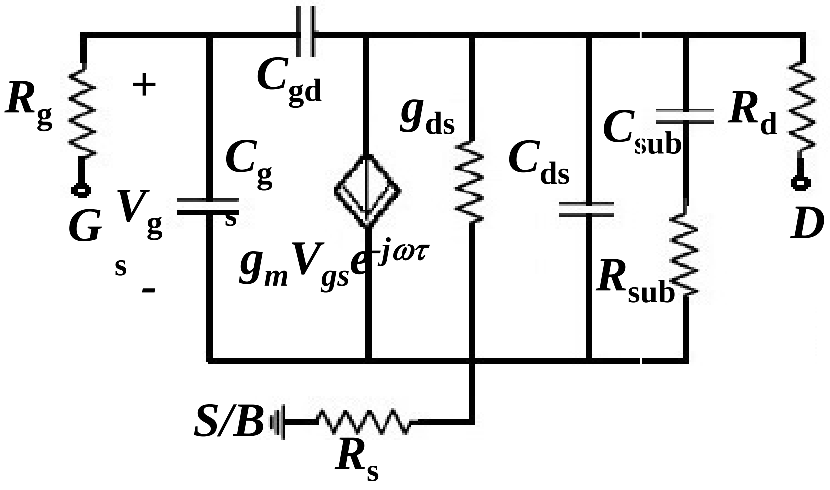
InP DHBT S21



InP DHBT S12

Commonly Used Small-signal Model for Transistors

- **Model normally extracted at hi-frequency, e.g. No thermal effect considered;**
- **For FinFETs application, the frequency range of thermal effect can high to ~1GHz, strong thermal effect can not be omitted in device modeling.**



Small-signal model for Si-FET devices.

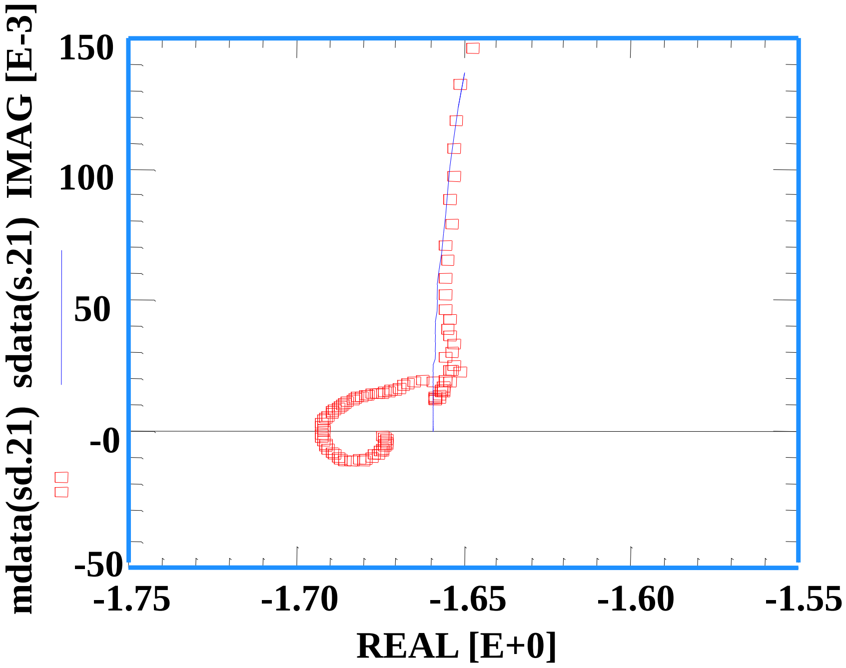


Fig. 1 Comparison of the model simulated (solid lines) and measured (symbols) *Y*-parameters (Y_{21}) of a FinFET device at $I_{be} = 2\times10^{-4}$A, $V_{ce} = 1.2$ V.

Context

Derivation of the Network Parameters with Thermal Effect

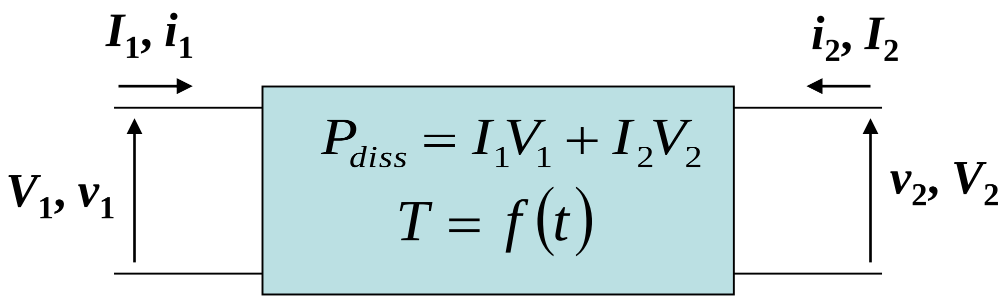


Fig.2 Voltages and currents in a two-port network.

$$I_1 = f_1(V_1, V_2, T)$$

$$I_2 = f_2(V_1, V_2, T)$$

$$T = T_{nom} + T_{rise} + V_t$$

$$V_t = Z_{th} P_{diss}$$

$$y_{mn} = \frac{dI_m}{dV_n} = \underbrace{\frac{\partial f_m}{\partial V_n}}_{\color{red}{P1}} + \underbrace{\frac{\partial f_m}{\partial T}}_{\color{red}{P2}} \underbrace{\frac{dT}{dV_n}}_{\color{red}{P3}} \Rightarrow$$

$$y^*_{mn} = \frac{\partial f_m}{\partial V_n} \qquad \frac{\partial f_m}{\partial T} = M_m I_m$$

M_m : Relative changes of the I_m per degree temperature change at constant voltages.

P1: Thermal Isolated Part, $T = T_{nom}$

P2: Instance Temperature Dependent Part, $T = T_{nom} + T_{rise}$

P3: Self-heating Dependent Part, $T = T_{nom} + Z_{th} * P_{diss}$

Y-parameters Accommodating Self-heating Behavior

$$y_{mn}=\frac{dI_m}{dV_n}=\underbrace{\frac{\partial f_m}{\partial V_n}}_{y^*_{mn}}+\underbrace{\frac{\partial f_m}{\partial T}}_{M_m I_m}\underbrace{\frac{dT}{dV_n}}_{\textbf{P3}}$$

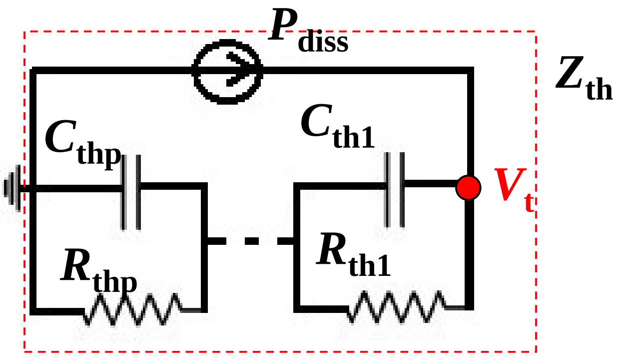


Fig.3 General thermal equivalent circuit

$$Z_{th}=\sum_{k=1}^{p}Z_{thk}=\sum_{k=1}^{p}R_{thk}/(1+j\omega R_{thk}C_{thk})$$

For P3:

$$dT=Z_{th}dP_{diss}$$

$$P_{diss}=I_1V_1+I_2V_2$$

$$\boxed{\frac{dT}{dV_n}=Z_{th}\frac{dP_{diss}}{dV_n}}$$

$$dP_{diss}=I_1dV_1+V_1dI_1+I_2dV_2+V_2dI_2$$, for y_{m1}, $dV_2= 0$; for y_{m2}, $dV_1 = 0$

$$\Rightarrow\quad y_{mn}=\frac{y^*_{mn}+M_mZ_{th}I_mI_n}{1-M_mZ_{th}P_{diss}}$$ **(Ref. O. Mueller, 1964)**

Compact Model Development

$$y_{mn} = \frac{y^*_{mn} + M_m Z_{th} I_m I_n}{1 - M_m Z_{th} P_{diss}}$$

Only DC consideration: $D_{ij} = -M_i \left(P_{diss} I_i / V_i + I_i I_j\right)$

$$y^*_{mn} = y_{mn} - Z_{th} \boxed{M_m \left(y_{mn} P_{diss} + I_m I_n\right)} = y_{mn} + \underline{Z_{th} D_{mn}}$$

$$Z_{th} = \sum_{k=1}^{p} Z_{thk} = \sum_{k=1}^{p} \left(R_{thk}^{-1} + j\omega R_{thk} C_{thk}\right)^{-1}$$

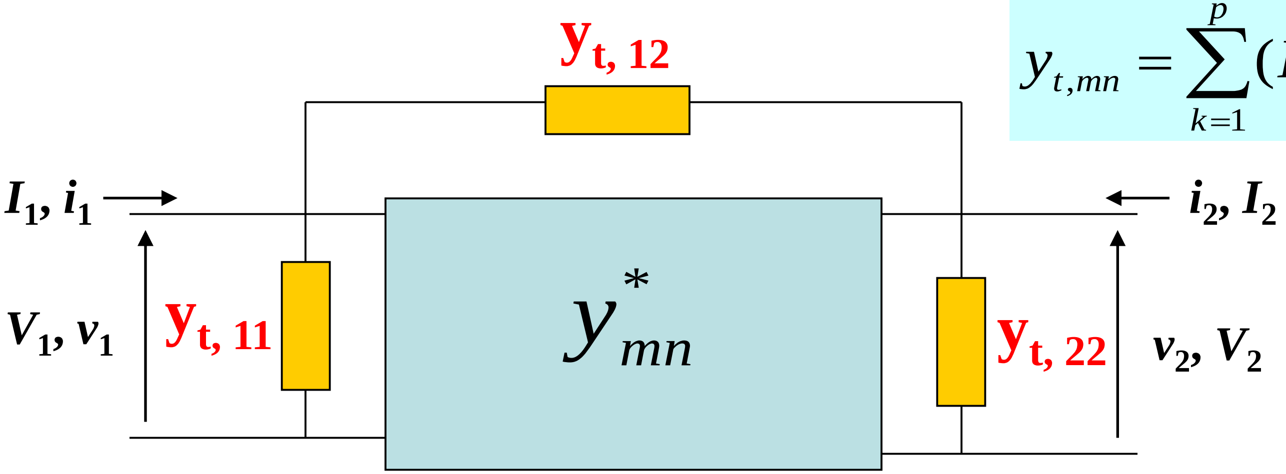
$$Z_{th} D_{mn} = D_{mn} \sum_{k=1}^{p} Z_{thk} = \sum_{k=1}^{p} \left[\underbrace{\boxed{\left(D_{mn} R_{thk}\right)^{-1}}}_{R^*_{mnk}} + j\omega \underbrace{\boxed{C_{thk} D_{mn}^{-1}}}_{L^*_{mnk}} \right]^{-1}$$

$$\Rightarrow \quad y^*_{mn} = y_{mn} + \sum_{k=1}^{p} \left(R^*_{mnk} + j\omega L^*_{mnk}\right)^{-1}$$

Enables complete small-signal model for transistors.

Small-signal Model Topology Development

$$y_{t,mn} = \sum_{k=1}^{p} (R^{*}_{mnk} + j\omega L^{*}_{mnk})^{-1}$$



$$y^{*}_{21} = y_{21} + \sum_{k=1}^{p} (R^{*}_{21k} + j\omega L^{*}_{21k})^{-1}$$

$$\Downarrow$$

$$g^{*}_{m} = g_{m} + \sum_{k=1}^{p} (R^{*}_{21k} + j\omega L^{*}_{21k})^{-1}$$

Fig.4 Complete small-signal model for transistors with thermal feedback caused by self-heating effect.

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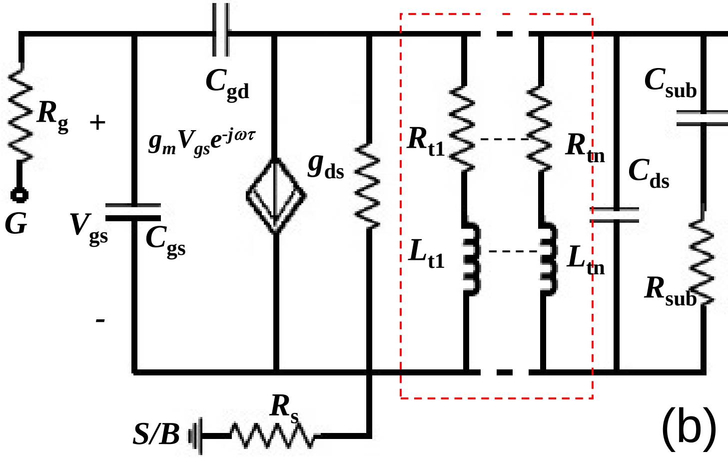
FinFET Devices Modeling

- **For FinFET devices, $I_1 = I_{gate} < 1.0e\text{-}9$ A, self-heating is mainly caused by $I_2 = I_{ds}$**

dP_{diss} can be simplified to $dP_{diss} = I_2 dV_2 + V_2 dI_2$

y_{t_11} and y_{t_12} are then can be omitted.

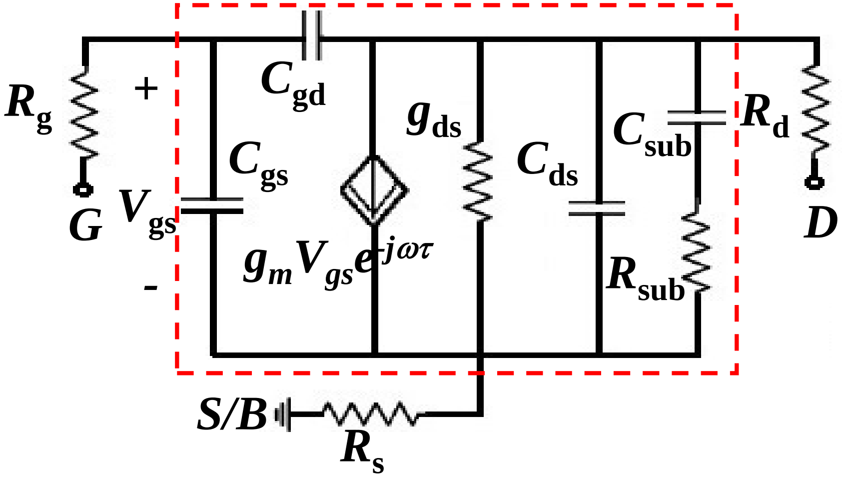


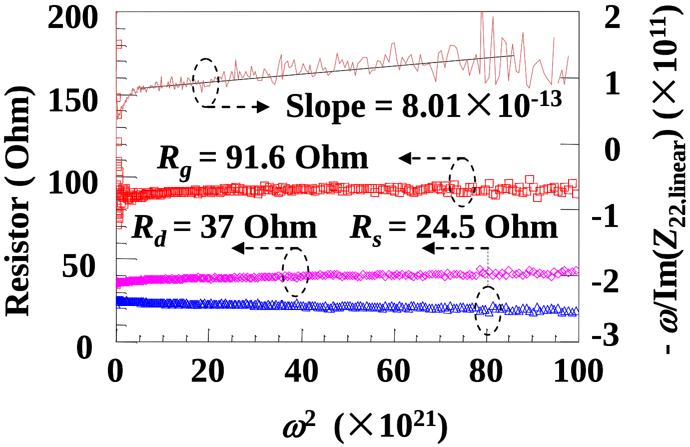
$$g_m = g_m^*\left[1+\sum_{i=1}^{n} k_{shi}/(1+j\omega\tau_{shi})\right] \quad k_{shi} = -M_2 P_{diss} R_{thi},\ \tau_{shi} = R_{thi} C_{thi},$$

Figure 5. Proposed small-signal model for FinFETs

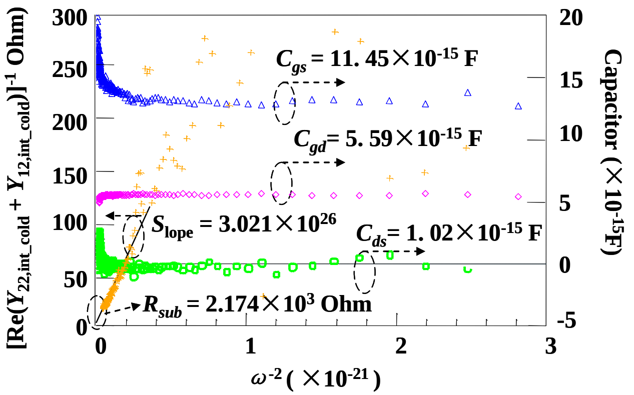
Model Parameter Extraction – HF Elements

- ☐ **Self-heating effect is a low frequency behavior, HF elements can be extracted from $Y|_{HF}$ at first, the thermal feedback elements are then can be extracted from low frequency *Y*-parameters, $Y|_{LF}$, analytically.**





- ➢ **R_s, R_s and R_g are extracted by using the methods proposed in [10]. Bias condition V_{ds} = 0 V and V_{gs} = 0.9 V is used.**
- ➢ **Once the terminal elements are extracted, the inner y-parameters are used to extract inner elements.**



Thermal Feedback Elements Extraction

- ☐ **dY= Y|$_{\mathbf{HF}}$(2, 2) – Y|$_{\mathbf{LF}}$ (2, 2) =** $\sum_{k=1}^{p}(R_{22k}^{*}+j\omega L_{22k}^{*})^{-1}$
- ☐ **dY= Y|$_{\mathbf{HF}}$(2, 1) – Y|$_{\mathbf{LF}}$ (2, 1) =** $\sum_{k=1}^{p}(R_{21k}^{*}+j\omega L_{21k}^{*})^{-1}$

TABLE I

Initially Extracted and Optimized Values of the Model Parameters at Bias 1: Vgs =0.6 V, Vds =0.9 V. RMS Error = |Extracted Value – Optimized Value| / Extracted Value ×100%. Model Parameters for SHE Characterization Only are Marked with Symbol *.

Parameter	Extracted Value	Optimized Value	Error (%)
$R_{\mathrm{g/d/s}}$ (Ω)	91.6, 37, 24.5	94.9, 39.8, 21.4	3.6, 7.5, 12.6
R_{sub} (kΩ), τ (pS)	2.295, 1.06	2.21, 0.92	4.1, 13.2
$C_{gs/d}$ (fF)	11.45, 5.5911.46, 5.390.8, 3.5		
C_{ds}, C_{sub} (fF)	1.02, 1.23	0.91, 1.34	10.7, 8.9
$^{*}R_{\mathrm{t1/2/3}}$ (kΩ)	204, 7.5, 8.4	198, 7.4, 8.6	2.9, 1.3, 2.3
$^{*}L_{\mathrm{t1/2/3}}$ (uH)	59k, 90, 24.6	57k, 87.6, 23.2	3.4, 2.7, 5.7
g_{ds}, g_{m} (mS)	0.72, 28.1	0.71, 25.82	1.3, 8.1
$^{*}\tau_{sh1/2/3}$ (nS)	290, 12.1, 2.9	266, 12.9, 2.7	8.2, 6.6, 6.9
$^{*}k_{sh1/2/3}$ ($\times 10^{-3}$)	-13, 28.3, 21.6	-14.2, 26.5, 22.1	9.2, 6.3, 2.3

Model Verification

☐ **Bulk FinFET, 4-Fin N-MOSFET with a total number of fingers Nf = 32 and gate length L = 16 nm, is fabricated using SMICs 14 nm bulk standard technology .**

☐ **A roughly frequency, f_{sh} = 0.75 GHz can be observed in the image parts of Y_{21} and Y_{22} where self-heating effect is negligible when freq > f_{sh}.**

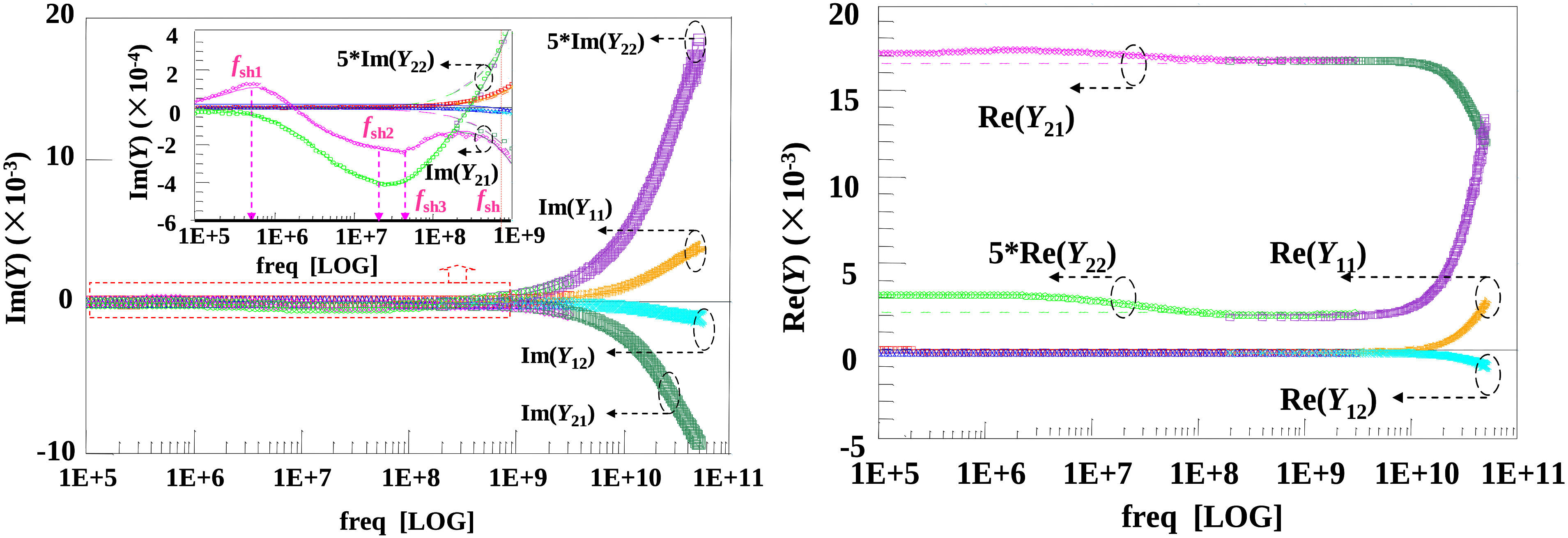


Fig. 6. Comparison of the model simulated with (solid lines) / without (dashed lines) SHE and measured (symbols) Y-parameters at V_{gs} = 0.6 V, Vds = 0.9 V.

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Summary and Discussion

- **A novel small-signal model for transistors accommodating self-heating behavior is presented.**
- **Based on the model theory, a method to accurately extract the thermal model of transistors is proposed.**
- **Discussion: Is the thermal model can be extracted from Pulse IV?**
  - **Thermal isolated testing: the time width of the pulse must less than 1/(2*pi*f_{sh});**
  - **R_{thi} and C_{thi} can not be extracted from Pulse IV.**

TABLE I

Extracted FinFET thermal Model Parameters at Bias: Vgs =0.6 V, Vds =0.9 V.

Parameter	Extracted Value	Optimized Value	Error (%)
$^{*}\tau_{sh1/2/3}$ (nS)	290, 12.1, **2.9**	266, 12.9, 2.7	8.2, 6.6, 6.9

Thanks !